$$I = \sum_{\text{ext spins}} \left| \left\langle out \middle| \hat{T} \middle| in \right\rangle \right|^2 = \sum_{\text{ext spins}} \left\langle out \middle| \hat{T} \middle| in \right\rangle \left\langle in \middle| \hat{T}^* \middle| out \right\rangle$$

Define the spin density matrix ρ such that:

$$|in\rangle\langle in| = \sum_{i,j} \rho_{i,j} = \sum_{i,j} |i\rangle\langle j|$$

Now

$$I = \sum_{\text{ext spins}} \sum_{i,j} \left\langle out \middle| {}^i\hat{T}\rho_{i,j}{}^j\hat{T}^* \middle| out \right\rangle$$

Let $\hat{T} = \hat{T}_d\hat{T}_p$, where subscript $d \equiv$ decay and $p \equiv$ production

$$I = \sum_{\text{ext spins}} \sum_{i,j} \left\langle out \middle| \hat{T}_d^i\hat{T}_p\rho_{i,j}{}^j\hat{T}_p^*\hat{T}_d^* \middle| out \right\rangle$$

Insert

$$1 = \sum_X |X\rangle\langle X|$$

and

$$1 = \sum_{X'} |X'\rangle\langle X'|$$

$$I = \sum_{\text{ext spins}} \sum_{i,j} \sum_{X,X'} \langle out| \hat{T}_d |X\rangle \langle X|^i \hat{T}_p\rho_{i,j}{}^j\hat{T}_p^* |X'\rangle \langle X'| \hat{T}_d^* |out\rangle$$

$$I(\Omega,\Phi) = 2\kappa \sum_k \left\{ (1-P_\gamma) \left| \sum_{l,m} [l]^{(-)}_{m;k} Re[Z_l^m(\Omega,\Phi)] \right|^2 + (1-P_\gamma) \left| \sum_{l,m} [l]^{(+)}_{m;k} Im[Z_l^m(\Omega,\Phi)] \right|^2 + (1+P_\gamma) \left| \sum_{l,m} [l]^{(+)}_{m;k} Re[Z_l^m(\Omega,\Phi)] \right|^2 + (1+P_\gamma) \left| \sum_{l,m} [l]^{(-)}_{m;k} Im[Z_l^m(\Omega,\Phi)] \right| \right\} \quad (1)$$

If $P = 0$

$$I(\Omega,\Phi) = 2\kappa \sum_k \left\{ \left| \sum_{l,m} [l]^{(-)}_{m;k} Z^m_l(\Omega,\Phi) \right|^2 + \left| \sum_{l,m} [l]^{(+)}_{m;k} Z^m_l(\Omega,\Phi) \right|^2 \right\} \tag{2}$$

Let $r = (-), (+)$ then

$$I(\Omega,\Phi) = 2\kappa \sum_{k,r} \left\{ \left| \sum_{l,m} [l]^{r}_{m;k} Z^m_l(\Omega,\Phi) \right|^2 \right\} \tag{3}$$