Sky mapping straight pipes

Sky mapping a pipe coincident with origin.

For a pipe, we can imagine sending a ray of light down the center ⇒ The pipe will map as the light does. If the light passes through the origin, the angles change as usual: $\tan\left(\frac{\theta'}{2}\right) = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} \tan\left(\frac{\theta}{2}\right)$

Sky mapping a pipe <u>not</u> coincident with origin.

In this case, sending light through the pipe is not useful. In this case, imagine the full length of pipe is luminescent. Trace the light from the pipe to the origin. Those rays, from pipe to origin, defines a plane that can be extended to pierce the unit (sky mapping) sphere.

Example:

* Boost in z
* Pipe parallel with y'
* pipe pierces x'-z' plane at $(x'=2, z'=2)$

⇒ plane of light: $x' = z'$

$$\Rightarrow \frac{\sin\theta\cos\phi}{\gamma - \gamma\beta\cos\theta} = \frac{\gamma\cos\theta - \gamma\beta}{\gamma - \gamma\beta\cos\theta} \Rightarrow \frac{x}{\gamma - \gamma\beta z} = \frac{\gamma z - \gamma\beta}{\gamma - \gamma\beta z}$$

$$\Rightarrow \quad x = \gamma z - \gamma\beta$$

The plane does <u>not</u> run through the origin when sky-mapped to frame S. In S, we see an arc of a circle instead of a straight line.