Partial wave analysis studies with simulated $\eta^{(\prime)}\pi^0$ events in GlueX

Florida International University 2020

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Outline

1. Introduction

- Evidence for π_1 (1600) exotic meson with π beam
- Search for exotic π_1 (1600) via $\gamma p \to p\eta'\pi^0$ in GlueX via Partial Wave Analysis (PWA), model for Intensity

2. Developing and testing methods for PWA using AMPTOOLs package

- Fitting generated $\gamma p \to p\eta^{(\prime)}\pi^0$ data sample to extract partial wave in different invariant mass and momentum transfer bins.
- Calculating moments of angular distribution using fitted partial waves.
- Extract unnormalized moment distributions using Monte Carlo integration
- Compare moment distributions obtained from both methods

3. Summary

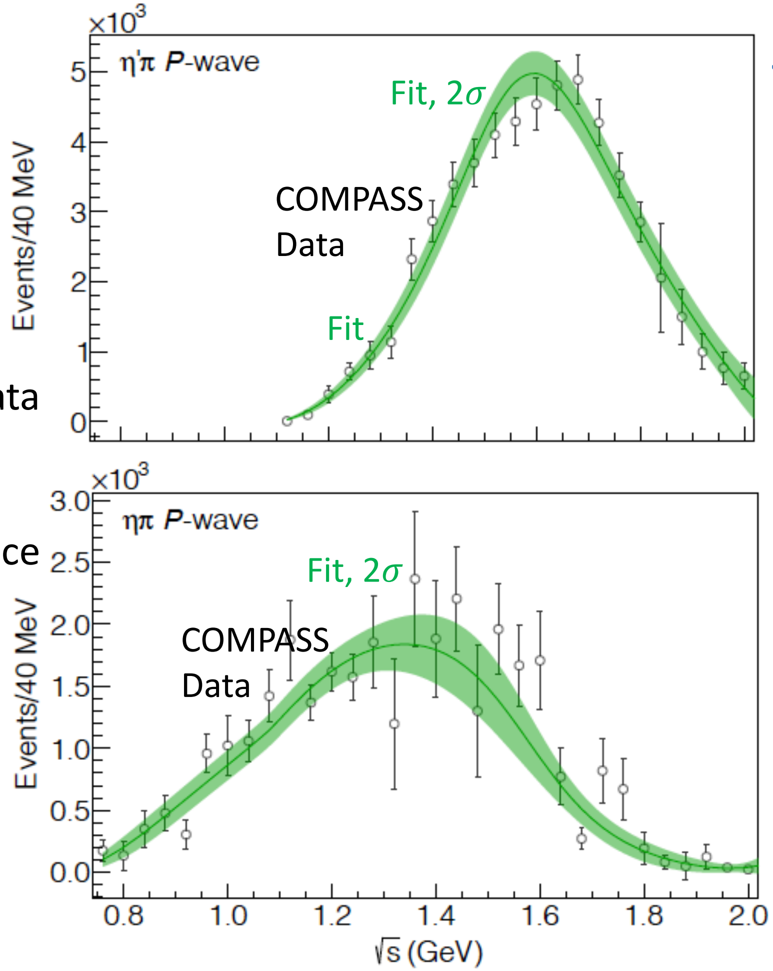
π_1 (1600) results from studies of $\eta'\pi$ system with π beam incident on a p target

Evidence for exotic $I^G J^{PC} = 1^-1^{-+}$ state π_1 (1600) produced via natural parity exchange (exchanged particle with J^Ps of 0+,1-,2+…)
G = C · $(-1)^I$, C operator followed by a rotation in isospin (I)

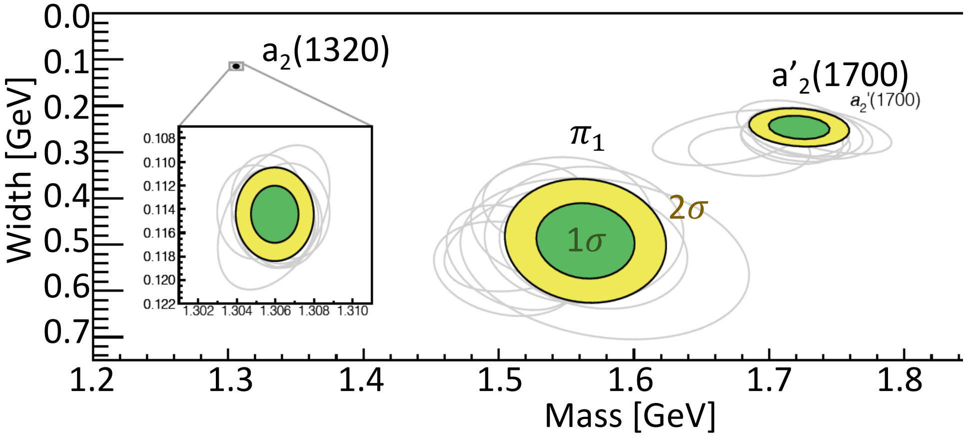
Several experiments suggest existence of π_1 from studies of $\eta'\pi$ system:

- **VES,** $E_\pi = 37$ GeV/c (D. V. Amelin et al., Phys. Atom. Nucl. 68, 359 (2005))
- **E852,** $E_\pi = 18$ GeV/c (E. I. Ivanov et al. [E852 Collaboration], Phys. Rev. Lett. 86, 3977 (2001))
- **COMPASS,** $E_\pi = 191$ GeV/c (C. Adolph, et al. [COMPASS Collaboration], Phys. Lett. B740, 303 (2015))

- Fit COMPASS data with amplitude model
- Extract resonance poles



Joint Physics Analysis Center



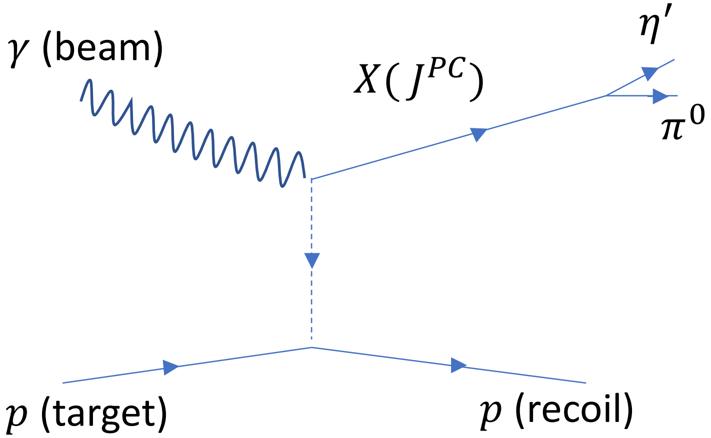
A. Rodas et al. [Joint Physics Analysis Center], PRL 122, 042002 (2019)

Search for exotic π_1 (1600) using the reaction $\gamma p \to p\eta'\pi^0$ in GLUEX

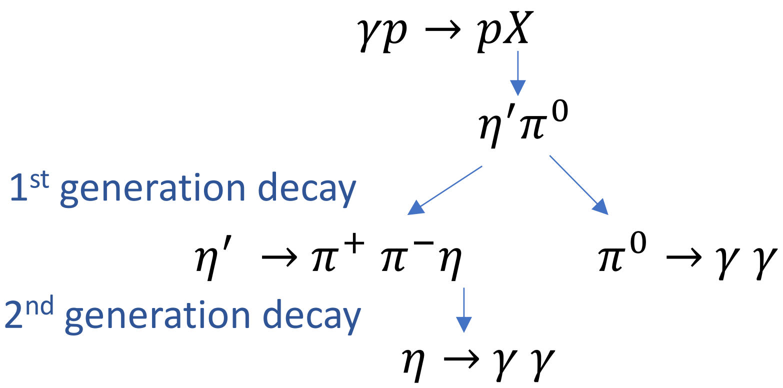
The odd waves in $\eta'\pi^0$ mesonic system have exotic quantum numbers and the lowest of them, the P-wave corresponds to exotic $\pi_1(1600)$ state.

GLUEX uses linearly polarized photon beam with E_γ~ 9GeV

$t -$ channel exchange



Decay mode



Model for Intensity with polarized photon beam in $\eta^{(\prime)}\pi^0$ photoproduction at GlueX

- Write the model predicted intensity I (number of signal events per unit phase space) in terms of partial wave amplitudes
- Introduce reflectivity basis to trade helicity λ for the reflectivity index $\epsilon=\pm1$ and write intensity as four coherent sums for each configuration of nucleon spin (spin flip k=1 and spin non-flip k=0)

Partial wave amplitudes (production of the wave)

Decay into two pseudo-scalars (parity constraints, L cons.)

$$I(\Omega,\Phi) = 2\kappa\sum_k\Bigg\{(1-P_\gamma)\Big|\sum_{l,m}[l]^{(-)}_{m;k}\mathrm{Re}[Z^m_l(\Omega,\Phi)]\Big|^2 + (1-P_\gamma)\Big|\sum_{l,m}[l]^{(+)}_{m;k}\mathrm{Im}[Z^m_l(\Omega,\Phi)]\Big|^2$$
$$+ (1+P_\gamma)\Big|\sum_{l,m}[l]^{(+)}_{m;k}\mathrm{Re}[Z^m_l(\Omega,\Phi)]\Big|^2 + (1+P_\gamma)\Big|\sum_{l,m}[l]^{(-)}_{m;k}\mathrm{Im}[Z^m_l(\Omega,\Phi)]\Big|^2\Bigg\}$$

l, m-orbital angular momentum and its projection

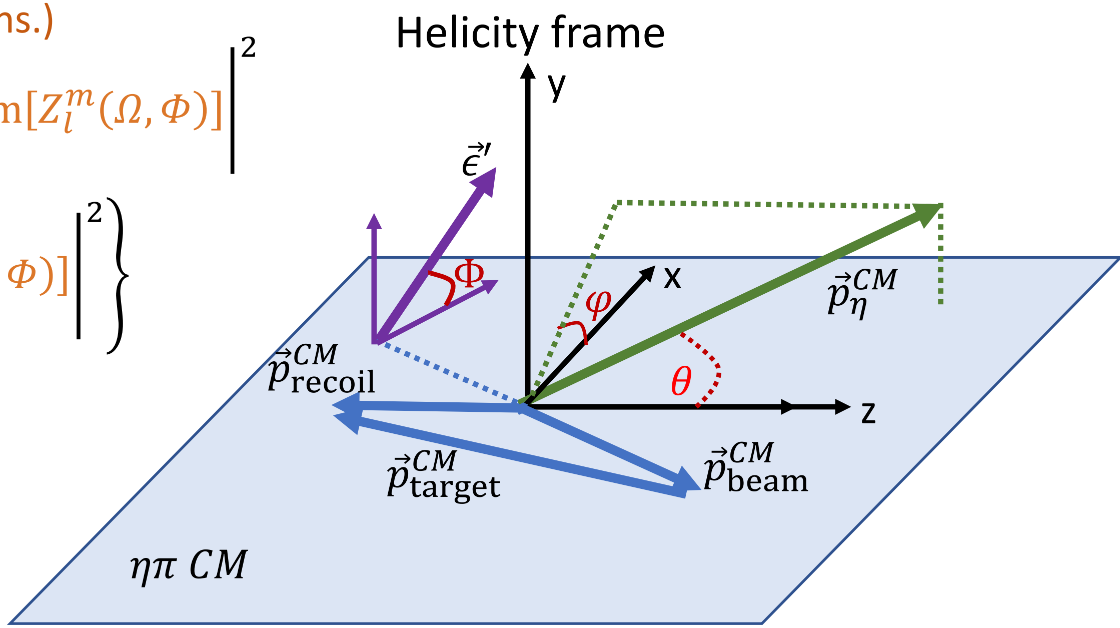
Φ - angle between γ polarization vector $\vec{\epsilon}'$ and production plane

$\Omega = (\theta,\varphi)$- direction of η in helicity frame

P_γ is the degree of linear polarization

$Z^m_l(\Omega,\Phi) \equiv Y^m_l(\Omega)e^{-i\Phi}$ are phase rotated spherical harmonics

κ- contains all kinematical factors



$\eta\pi$ partial wave is characterized by $J^{PC}M^{\epsilon}$

Looking for $J^{PC} = 1^{-+}$ partial wave.

- Bin data in small bins of $m_{\eta\pi}$, t and E_γ with constant $[l]^{(-)}_{m;k}$, $[l]^{(+)}_{m;k}$
- Fit data using extended unbinned (in (θ,φ)) maximum likelihood method

$$\ln L(l) = \sum_{i=1}^{N}\ln I(l,\theta,\varphi) - \int I(l,\theta,\varphi)\eta(\theta,\varphi)\,d\Omega$$

$\eta(\theta,\varphi)$ -acceptance

- Minimize –lnL using MINUIT, to find $[l]^{(-)}_{m;k}$, $[l]^{(+)}_{m;k}$

V. Mathieu et al. Phys. Rev. D 100, 054017 (2019)

Generated 2*10^6 $(p\eta\pi^0)$events with AmpTools

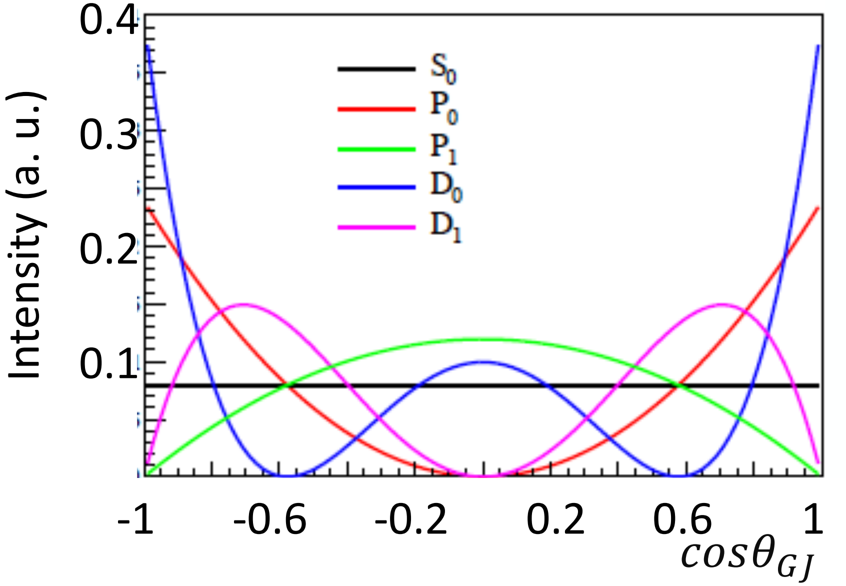
Generated resonances are

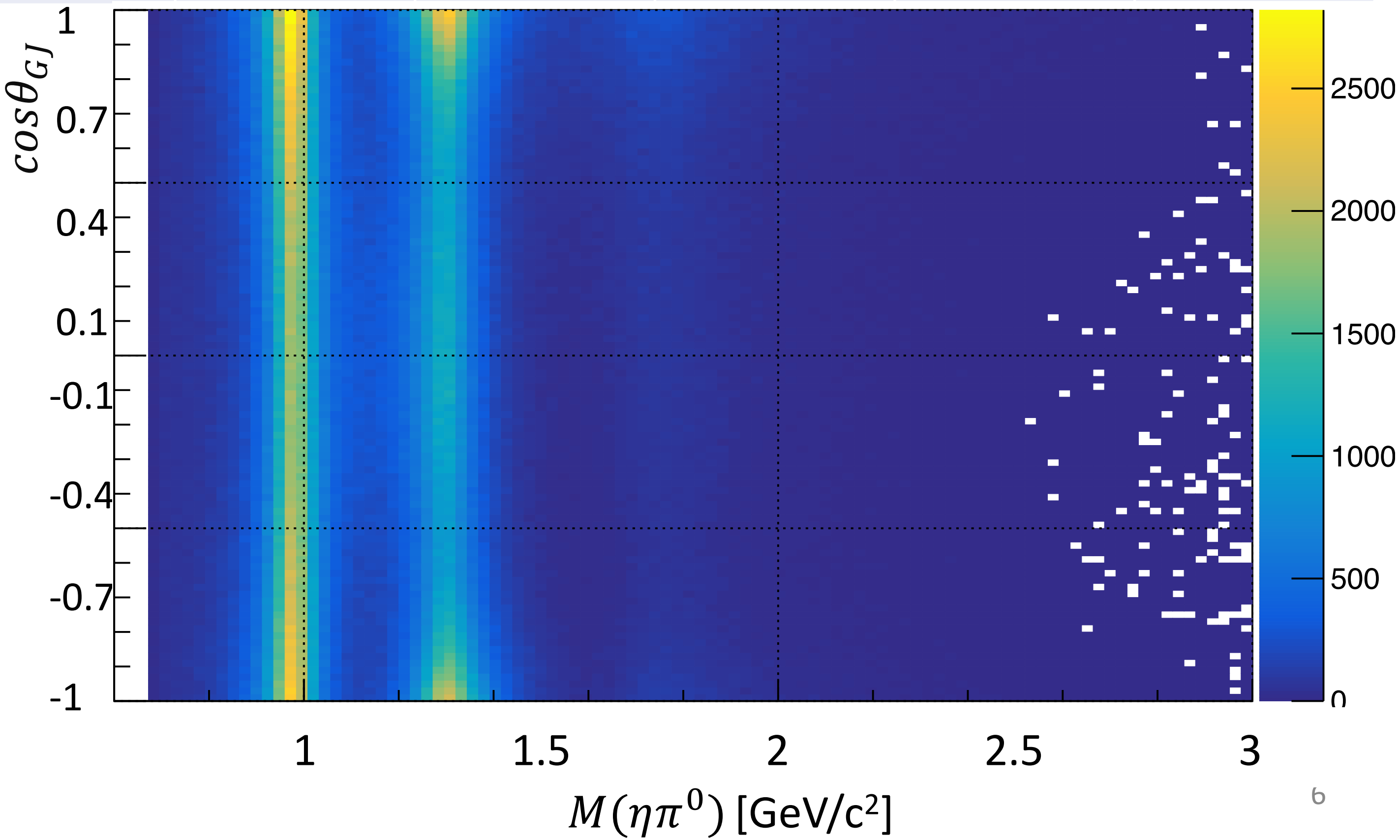
- a_0(980 MeV)
- π_1(1600 MeV) (exotic)
- a2 (1320 MeV)
- a2' (1700 MeV)

θ_{pol}=1.77 Deg.
P_γ =0.3

J	M	ε	Real	Imaginary	BW Mass	BW Width
0	0	+1	1000	0	0.980	0.075
1	0, 1	+1	70	70	1.564	0.492
2	0,1,2	+1	150	150	1.306	0.114
2	0,1,2	+1	50	50	1.722	0.247

The wave set: $[l]_{m;k}^{(\epsilon)} = \left\{S_0^{(+)}, P_{0,1}^{(+)}, D_{0,1,2}^{(+)}\right\}_{k=0}$ with M>= 0





Analysis strategy

For the wave set $[l]^{(\epsilon)}_{m;k} = \left\{S_0^{(+)}, P_{0,1}^{(+)}, D_{0,1,2}^{(+)}\right\}_{k=0}$ with M>= 0

1. Fit intensity to find partial waves using AmpTools.

2. Implement and test calculation of moments of angular distribution in terms of partial waves using the following expressions in terms of the $\eta'\pi^0$ SDMEs ${}^{(\epsilon)}\rho^{\alpha,ll'}_{mm'}$ calculated in reflectivity basis:

$$H^0(LM) = \sum_{\substack{ll' \\ mm'}} \left(\frac{2l'+1}{2l+1}\right)^{1/2} C^{l0}_{l'0L0} C^{lm}_{l'm'LM}\, \rho^{0,ll'}_{mm'}$$

$$\boldsymbol{H}(LM) = -\sum_{\substack{ll' \\ mm'}} \left(\frac{2l'+1}{2l+1}\right)^{1/2} C^{l0}_{l'0L0} C^{lm}_{l'm'LM}\, \boldsymbol{\rho}^{ll'}_{mm'}$$

$$\rho^{\alpha,ll'}_{mm'} = \sum_{\epsilon} {}^{(\epsilon)}\rho^{\alpha,ll'}_{mm'}$$

$${}^{(\epsilon)}\rho^{0,ll'}_{mm'} = \kappa \sum_k \left(|l|^{(\epsilon)}_{m;k} |l'|^{(\epsilon)*}_{m';k} + (-1)^{m-m'} |l|^{(\epsilon)}_{-m;k} |l'|^{(\epsilon)*}_{-m';k} \right)$$

$${}^{(\epsilon)}\rho^{1,ll'}_{mm'} = -\epsilon\kappa \sum_k \left((-1)^m |l|^{(\epsilon)}_{-m;k} |l'|^{(\epsilon)*}_{m';k} + (-1)^{m'} |l|^{(\epsilon)}_{m;k} |l'|^{(\epsilon)*}_{-m';k} \right)$$

$${}^{(\epsilon)}\rho^{2,ll'}_{mm'} = -i\epsilon\kappa \sum_k \left((-1)^m |l|^{(\epsilon)}_{-m;k} |l'|^{(\epsilon)*}_{m';k} - (-1)^{m'} |l|^{(\epsilon)}_{m;k} |l'|^{(\epsilon)*}_{-m';k} \right)$$

$${}^{(\epsilon)}\rho^{3,ll'}_{mm'} = \kappa \sum_k \left(|l|^{(\epsilon)}_{m;k} |l'|^{(\epsilon)*}_{m';k} - (-1)^{m-m'} |l|^{(\epsilon)}_{-m;k} |l'|^{(\epsilon)*}_{-m';k} \right)$$

where $C^{l0}_{l'0L0}$ and $C^{lm}_{l'm'LM}$ denote the Clebsch-Gordan coeficients, 0≤ $L \leq 4$ and 0≤ $M \leq L$.

Non-zero odd L moments ↔ presence of exotic wave. If M=0, H^2 and H^3 are 0.

3. Compare the results from previous step to moment distributions obtained by Monte Carlo integration based on the expressions:

$$H^0(LM) = \frac{P_\gamma}{2} \int_o I(\Omega,\Phi) d^L_{M0}(\theta) \cos M\phi,$$

$$H^1(LM) = \int_o I(\Omega,\Phi) d^L_{M0}(\theta) \cos M\phi \cos 2\Phi,$$

$$\mathrm{Im} H^2(LM) = -\int_o I(\Omega,\Phi) d^L_{M0}(\theta) \sin M\phi \sin 2\Phi,$$

with $\int_o = (1/\pi P_\gamma) \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi \int_0^{2\pi} d\Phi$ and $d^L_{M0}(\theta)$ denotes Wigner d-function.

4. Compare moments from fitting with true wave set (S0+, P0+,P1+,D0+,D1+,D2+) using good starting values for fit parameters (partial wave amplitudes $[l]^{(\epsilon)}_{m;k}$) to moments from:

- Fit 1 : fitting with S0+, P0+, D0+, D1+, G0+ ,G1+ waveset using good starting values for the fit parameters that are common
- Fit 2: fitting with S0-, P0+, P1+, D0+, D1+, D2+, G0+ ,G1+ waveset using good starting values for the fit parameters that are common

Mathieu et al. Phys. Rev. D 100, 054017 (2019)

Implementation of calculation of moments

1. Implement and test calculation of moments in terms of partial waves using the following expressions in terms of the $\eta'\pi^0$ SDMEs calculated in reflectivity basis:

$$H^0(LM) = \sum_{\substack{\ell\ell' \\ mm'}} \left(\frac{2\ell'+1}{2\ell+1}\right)^{1/2} C^{\ell 0}_{\ell' 0 L 0} C^{\ell m}_{\ell' m' L M}\ \rho^{\alpha,\ell\ell'}_{mm'} \qquad \rho^{\alpha,ll'}_{mm'} = \sum_{\epsilon} {}^{(\epsilon)}\rho^{\alpha,ll'}_{mm'}$$

$$H(LM) = -\sum_{\substack{\ell\ell' \\ mm'}} \left(\frac{2\ell'+1}{2\ell+1}\right)^{1/2} C^{\ell 0}_{\ell' 0 L 0} C^{\ell m}_{\ell' m' L M}\ \rho^{\ell\ell'}_{mm'}$$

$${}^{(\epsilon)}\rho^{0,\ell\ell'}_{mm'} = \kappa \sum_k \left([\ell]^{(\epsilon)}_{m;k} [\ell']^{(\epsilon)*}_{m';k} + (-1)^{m-m'} [\ell]^{(\epsilon)}_{-m;k} [\ell']^{(\epsilon)*}_{-m';k} \right),$$

$${}^{(\epsilon)}\rho^{1,\ell\ell'}_{mm'} = -\epsilon\kappa \sum_k \left((-1)^{m} [\ell]^{(\epsilon)}_{-m;k} [\ell']^{(\epsilon)*}_{m';k} + (-1)^{m'} [\ell]^{(\epsilon)}_{m;k} [\ell']^{(\epsilon)*}_{-m';k} \right),$$

$${}^{(\epsilon)}\rho^{2,\ell\ell'}_{mm'} = -i\epsilon\kappa \sum_k \left((-1)^{m} [\ell]^{(\epsilon)}_{-m;k} [\ell']^{(\epsilon)*}_{m';k} - (-1)^{m'} [\ell]^{(\epsilon)}_{m;k} [\ell']^{(\epsilon)*}_{-m';k} \right),$$

$${}^{(\epsilon)}\rho^{3,\ell\ell'}_{mm'} = \kappa \sum_k \left([\ell]^{(\epsilon)}_{m;k} [\ell']^{(\epsilon)*}_{m';k} - (-1)^{m-m'} [\ell]^{(\epsilon)}_{-m;k} [\ell']^{(\epsilon)*}_{-m';k} \right).$$

The calculation is implemented by me in "**project_moments_polarized**"
Another version of the calculation based on Vincents codes is called "**Pol_moments_viafittedPW**"

2. The calculation based on explicit formulas

$$H^0(00) = H^1(00) + 2\left[|P_1^{(+)}|^2 + |D_1^{(+)}|^2 + |D_2^{(+)}|^2\right]$$

$$H^1(22) = H^0(22) + \frac{\sqrt{6}}{7}|D_1^{(+)}|^2 + \frac{\sqrt{6}}{5}|P_1^{(+)}|^2$$

that is applicable for M>=0, ϵ>0 and L<=D is coded in "**project_moments_SPD_etapi0_posepsilon**".
All three codes can be found in **halld_sim/src/programs/AmplitudeAnalysis/**

3. I have also added scripts and codes for plotting moments in
hd_utilities/PWA_scripts/Polarized_moments_viaPW

Config file for fitting with generated amplitudes in M and t bins

```
define polVal 0.3
fit FITNAME
reaction EtaPrimePi0 Beam Proton Eta Pi0

genmc  EtaPrimePi0 ROOTDataReader GENMCFILE
accmc  EtaPrimePi0 ROOTDataReader ACCMCFILE
data EtaPrimePi0 ROOTDataReader DATAFILE

sum EtaPrimePi0 PositiveRe
sum EtaPrimePi0 PositiveIm
parameter polAngle 1.77 fixed

# a0(980)
amplitude EtaPrimePi0::PositiveIm::S0+ Zlm 0 0 -1 -1 polAngle polVal
amplitude EtaPrimePi0::PositiveRe::S0+ Zlm 0 0 +1 +1 polAngle polVal
# a2(1320)a2'(1700)
amplitude EtaPrimePi0::PositiveIm::D0+ Zlm 2 0 -1 -1 polAngle polVal
amplitude EtaPrimePi0::PositiveRe::D0+ Zlm 2 0 +1 +1 polAngle polVal
amplitude EtaPrimePi0::PositiveIm::D1+ Zlm 2 1 -1 -1 polAngle polVal
amplitude EtaPrimePi0::PositiveRe::D1+ Zlm 2 1 +1 +1 polAngle polVal
amplitude EtaPrimePi0::PositiveIm::D2+ Zlm 2 2 -1 -1 polAngle polVal
amplitude EtaPrimePi0::PositiveRe::D2+ Zlm 2 2 +1 +1 polAngle polVal
# pi1(1600)      • • •

initialize EtaPrimePi0::PositiveIm::S0+ cartesian 1000.0 0.0 real
initialize EtaPrimePi0::PositiveRe::S0+ cartesian 1000.0 0.0 real
initialize EtaPrimePi0::PositiveIm::D0+ cartesian 70.0 70.0
initialize EtaPrimePi0::PositiveRe::D0+ cartesian 70.0 70.0
initialize EtaPrimePi0::PositiveIm::D1+ cartesian 70.0 70.0
initialize EtaPrimePi0::PositiveRe::D1+ cartesian 70.0 70.0
initialize EtaPrimePi0::PositiveIm::D2+ cartesian 70.0 70.0
initialize EtaPrimePi0::PositiveRe::D2+ cartesian 70.0 70.0
      • • •

constrain EtaPrimePi0::PositiveIm::S0+ EtaPrimePi0::PositiveRe::S0+
      • • •
```

Typically refers to unique set of initial and final state particles
Can also refer to multiple decay modes of the same set of final state particles

Reaction, data reader class, argument

Events to fit intensity to

All amplitudes within a given sum are added coherently

User defined classes

Keywords

Reaction, Sum, amplitude name, amplitude class, arguments

```
Zlm as suggested in GlueX doc-4094 (M. Shepherd)
argument 1 : j
argument 2 : m
argument 3 : real (+1) or imaginary (-1) part
argument 4 : 1 + (+1/-1) * P_gamma
argument 5 : polarization angle (in Deg.)
argument 6 : beam properties config file or fixed polarization
```

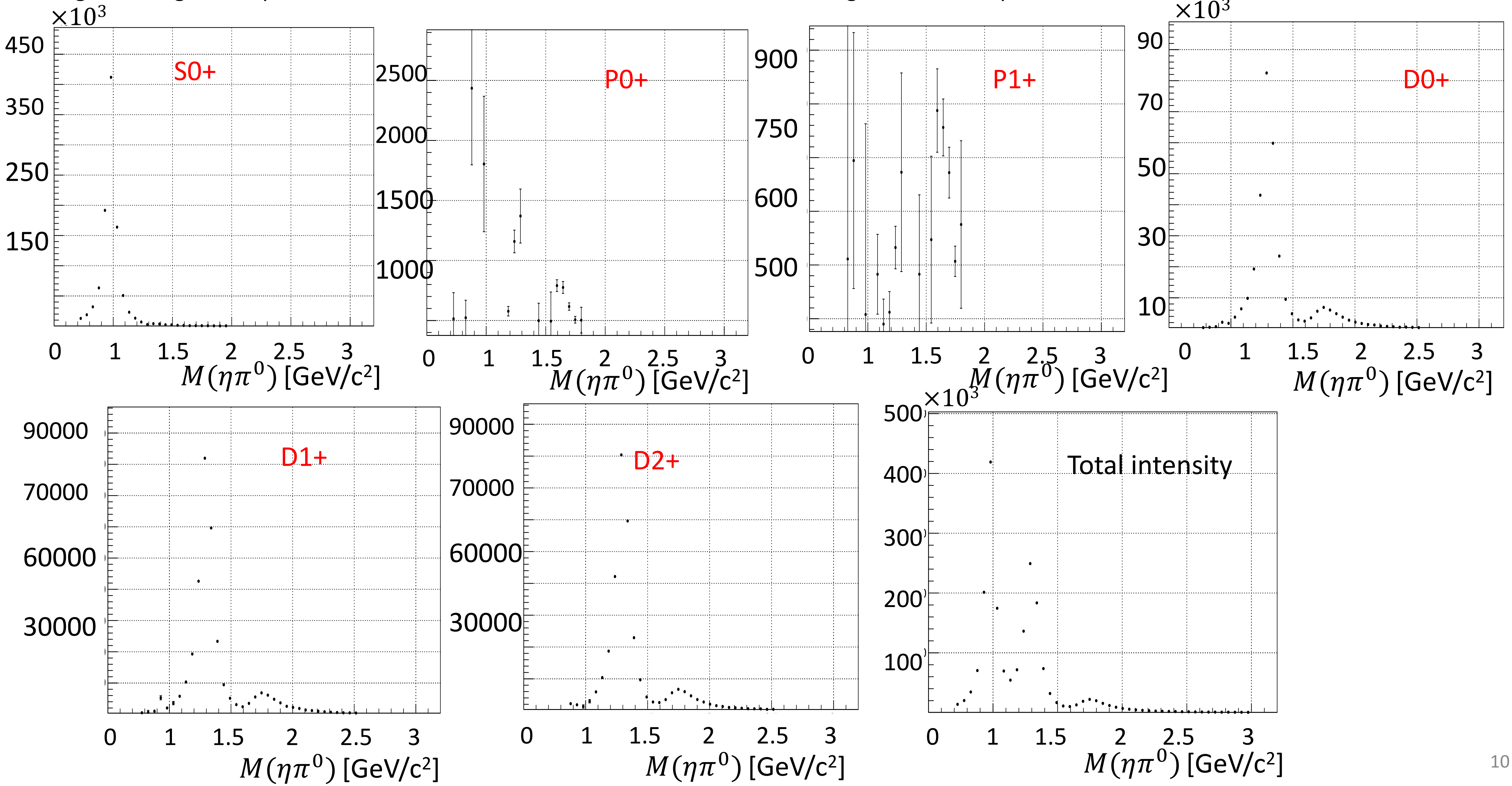
Initial value of partial wave amplitudes $[l]^{(\epsilon)}_{m;k}$ in cartesian coordinate system

Factors with the same reaction sum and amplitude name are multiplied together

Same amplitudes corresponding to different sums should be equal

Fit 1 results (fitting in M and t bins)

Fitting with original amplitude set: S0-, P0+, P1+, D0+, D1+, D2+. Good starting values for fit parameters.



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Codes and scripts for calculation and plotting of moments for N_M and N_t bins

1. First one needs to fit intensity with certain amplitude set in different M and t bins to obtain fitted amplitudes (see my talk from previous collab. meeting for details).

One can then calculate the moments for each M and t bin and write all to a file etaprimepi0_moments.txt via the following command:

project_moments_polarized -o etaprimepi0_moments.txt

One needs to edit the code and modify the list of the waves. It will look for fit results in ./etaprimepi0 directory.

The first line in the output file will contain the names of the variables in each colomn (M, t and moments)

2. To plot the moments as a function of M for the first t bin do:

python **Drawing_moments_M_t_bins.py** N_M N_t etaprimepi0_moments.txt

The graphs of moments will be written to a .root file and corresponding plots will be saved in Plots directory in current directory.

3. To plot two different results together do:

root -l **Plot_graphs_together.C**++

4. To obtain moment distributions by Monte Carlo integration (code from Rebecca) do:

root -l **plotMoments.C**++

Codes for calculating moments can be found in

halld_sim/src/programs/AmplitudeAnalysis/

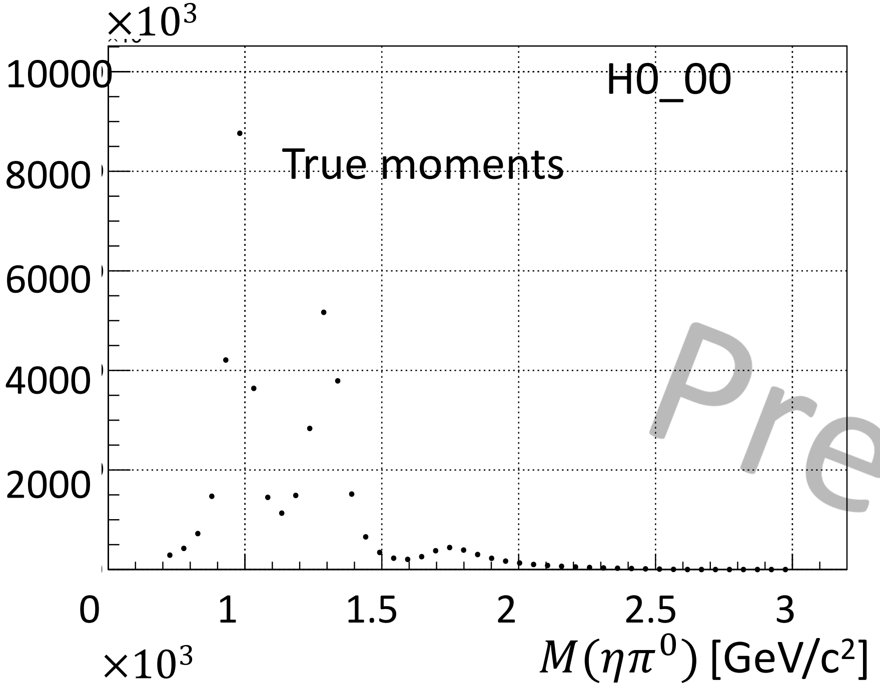
Codes for plotting can be found in

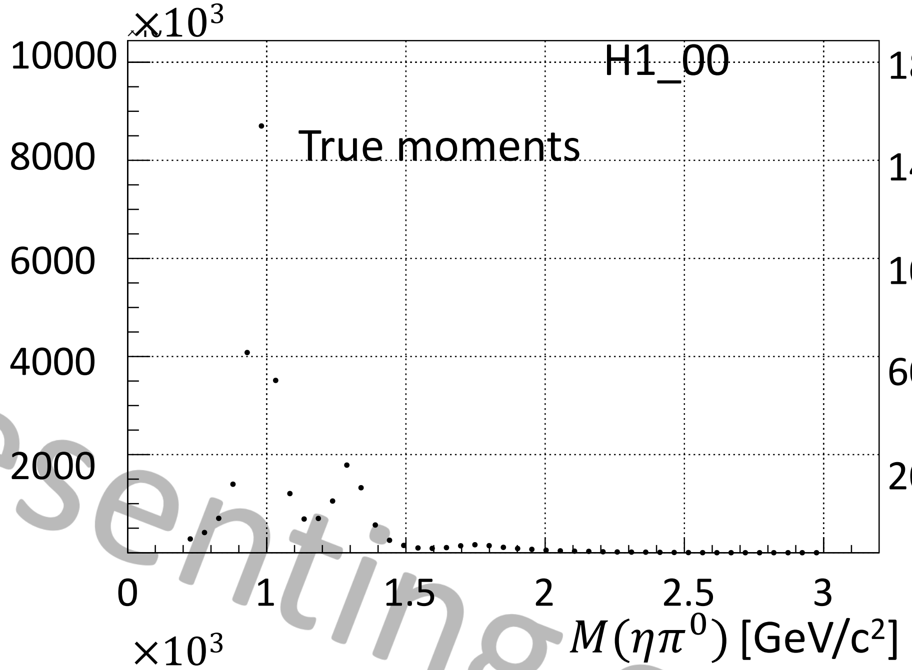
hd_utilities/PWA_scripts/Polarized_moments_viaPW /Plotting_polarized_moments/

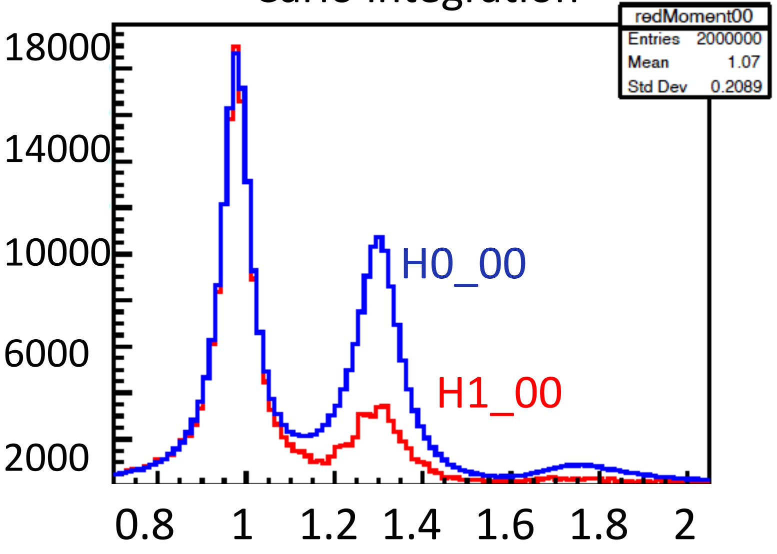
Polarized moments calculated with partial waves

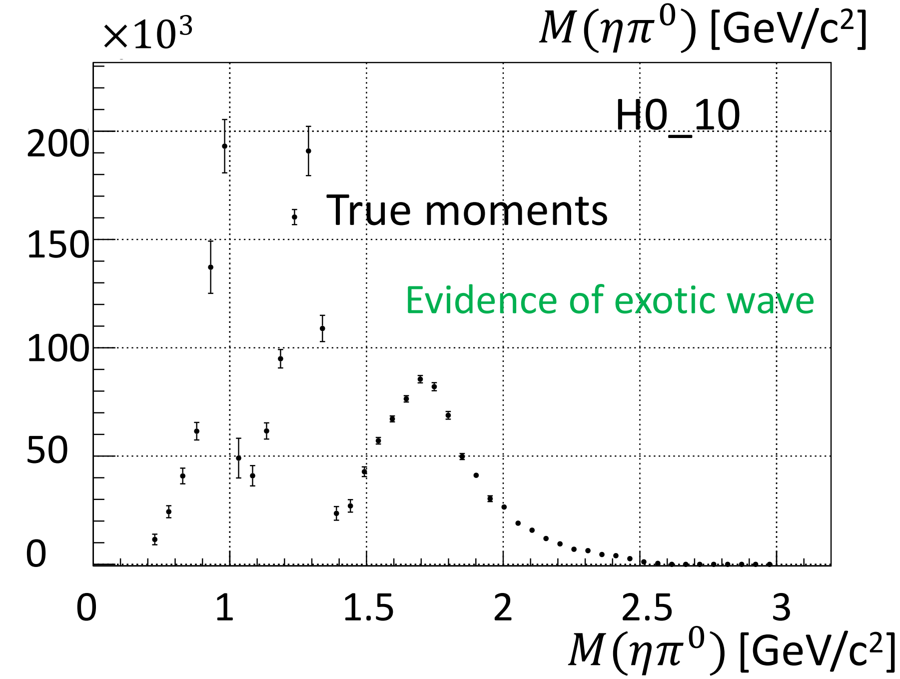
$0<t<0.3$ $(GeV/c)^2$ With uncertainties from bootstrapping

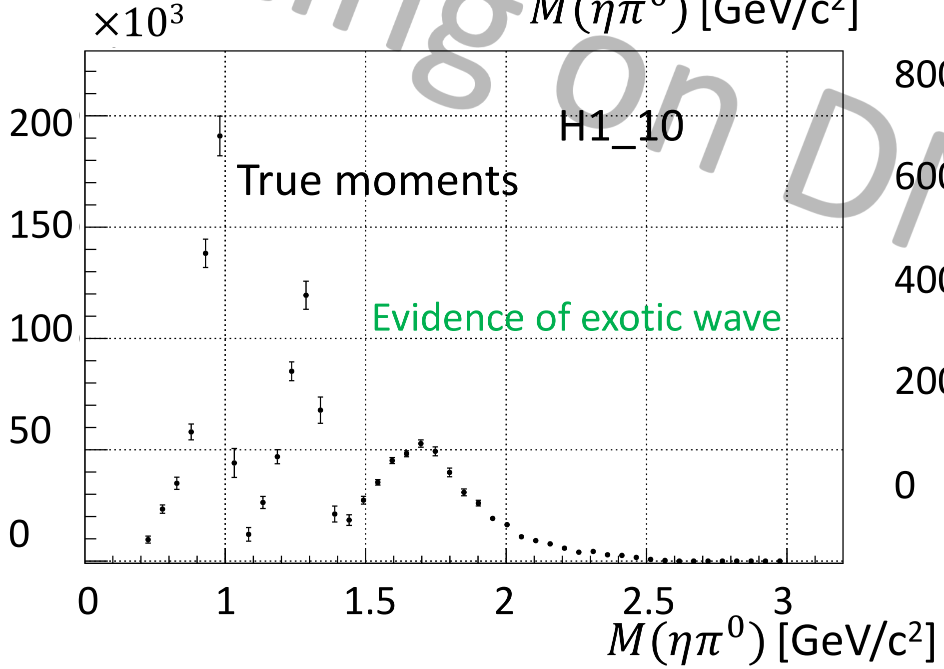
Unnormalized moments from Monte Carlo integration

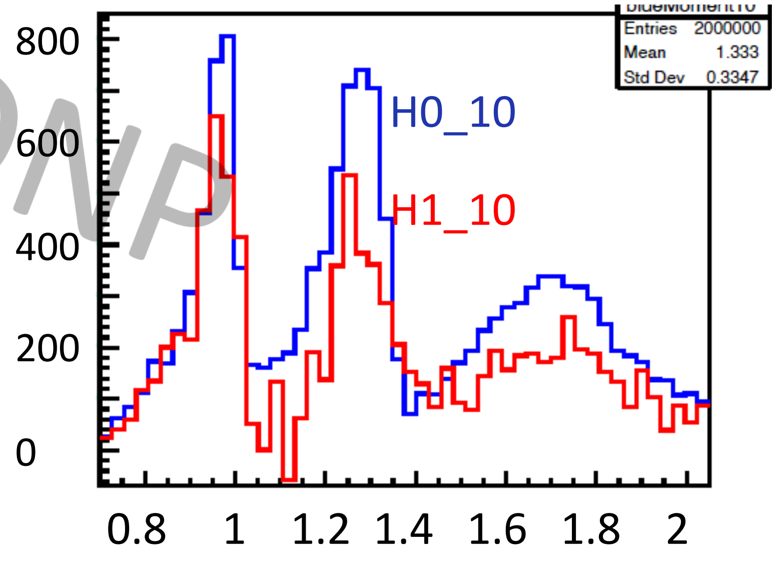












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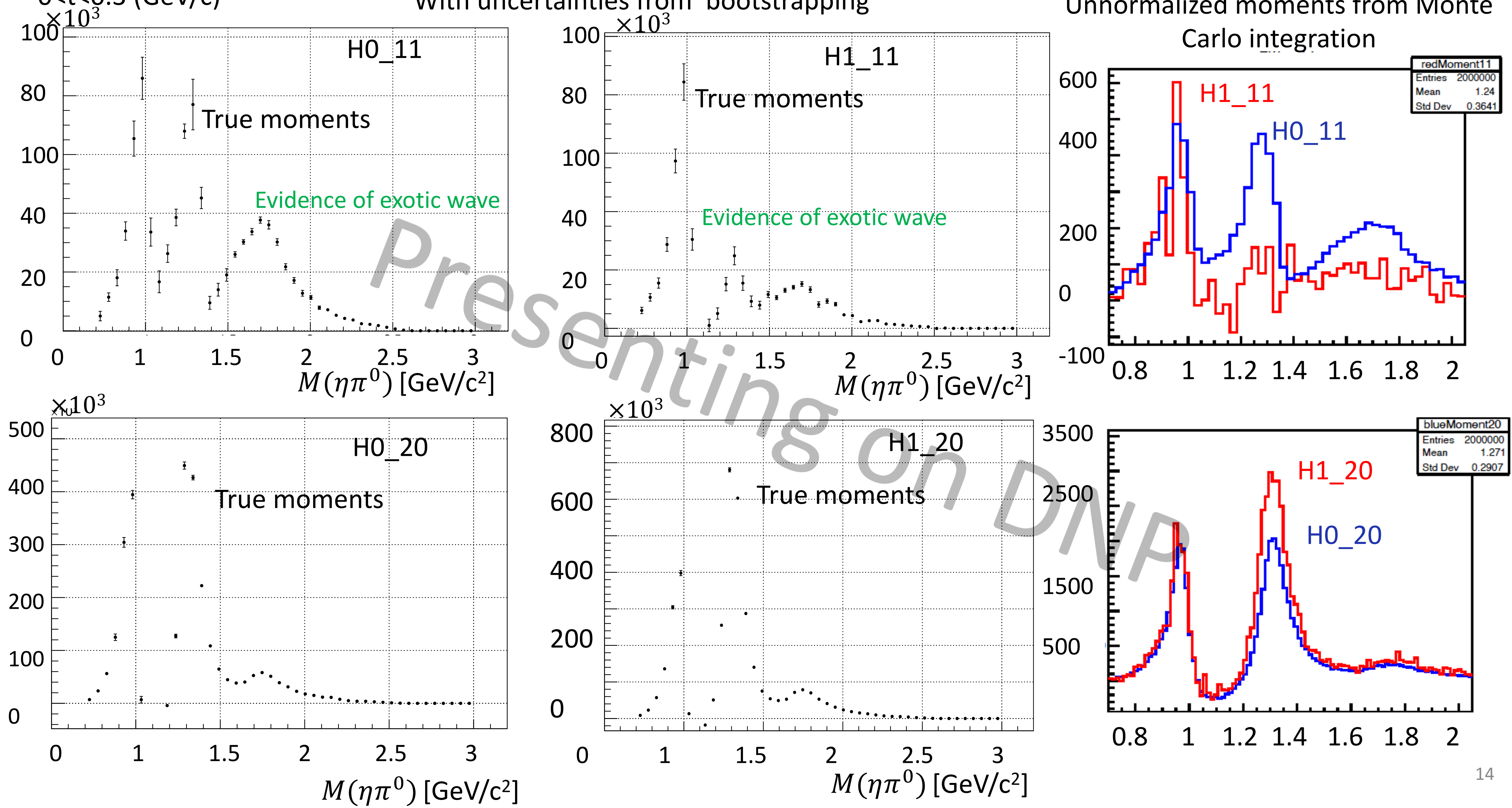
With uncertainties from bootstrapping

Unnormalized moments from Monte Carlo integration



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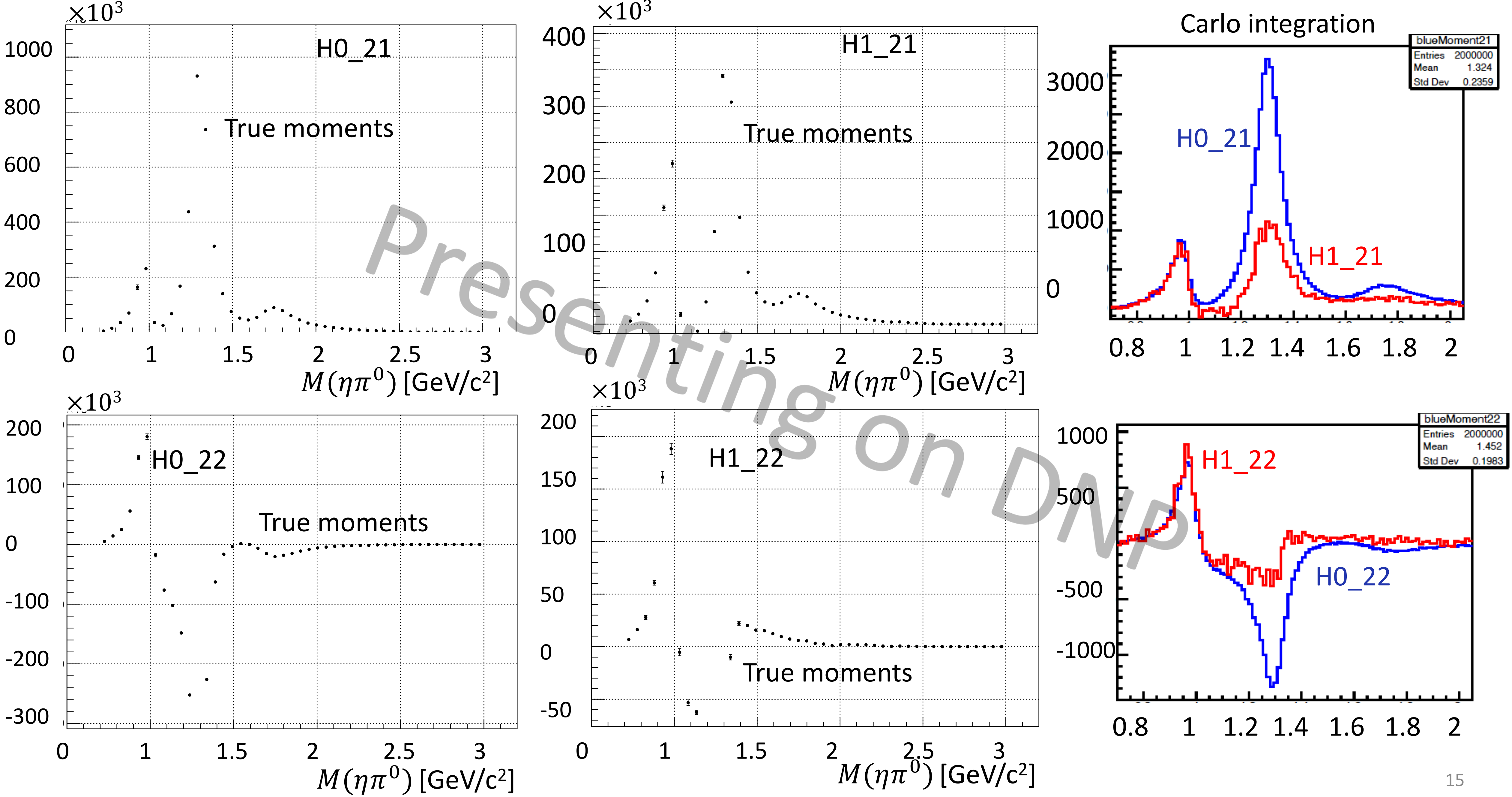

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$${}^{(\epsilon)}\rho^{1,ll'}_{mm'} = -\epsilon\kappa \sum_k \left((-1)^{m} |l|^{(\epsilon)}_{-m;k} |l'|^{(\epsilon)*}_{m';k} + (-1)^{m'} |l|^{(\epsilon)}_{m;k} |l'|^{(\epsilon)*}_{-m';k} \right)$$

$${}^{(\epsilon)}\rho^{2,ll'}_{mm'} = -i\epsilon\kappa \sum_k \left((-1)^{m} |l|^{(\epsilon)}_{-m;k} |l'|^{(\epsilon)*}_{m';k} - (-1)^{m'} |l|^{(\epsilon)}_{m;k} |l'|^{(\epsilon)*}_{-m';k} \right)$$

$${}^{(\epsilon)}\rho^{3,ll'}_{mm'} = \kappa \sum_k \left(|l|^{(\epsilon)}_{m;k} |l'|^{(\epsilon)*}_{m';k} - (-1)^{m-m'} |l|^{(\epsilon)}_{-m;k} |l'|^{(\epsilon)*}_{-m';k} \right)$$

where $C^{l0}_{l'0L0}$ and $C^{lm}_{l'm'LM}$ denote the Clebsch-Gordan coeficients, 0≤ $L \leq 4$ and 0≤ $M \leq L$.
Non-zero odd L moments ↔ presence of exotic wave. If M=0, H^2 and H^3 are 0.

3. Compare the results from previous step to moment distributions obtained by Monte Carlo integration based on the expressions:

$$H^0(LM) = \frac{P_\gamma}{2} \int_o \; I(\Omega,\Phi) d^L_{M0}(\theta) cosM\phi,$$

$$H^1(LM) = \int_o \; I(\Omega,\Phi) d^L_{M0}(\theta) cosM\phi\, cos2\Phi,$$

$$\mathrm{Im}H^2(LM) = -\int_o \; I(\Omega,\Phi) d^L_{M0}(\theta) sinM\phi\, sin2\Phi,$$

with $\int_o \; = (1/\pi P_\gamma) \int_0^\pi sin\theta d\theta \int_0^{2\pi} d\phi \int_0^{2\pi} d\Phi$ and $d^L_{M0}(\theta)$ denotes Wigner d-function.

4. Compare moments from fitting with true wave set (S0+, P0+,P1+,D0+,D1+,D2+) using good starting values for fit parameters (partial wave amplitudes) to moments from:
   - Fit 1 : fitting with S0+, P0+, D0+, D1+, G0+ ,G1+ waveset using good starting values for the fit parameters that are common
   - Fit 2: fitting with S0-, P0+, P1+, D0+, D1+, D2+, G0+ ,G1+ waveset using good starting values for the fit parameters that are common

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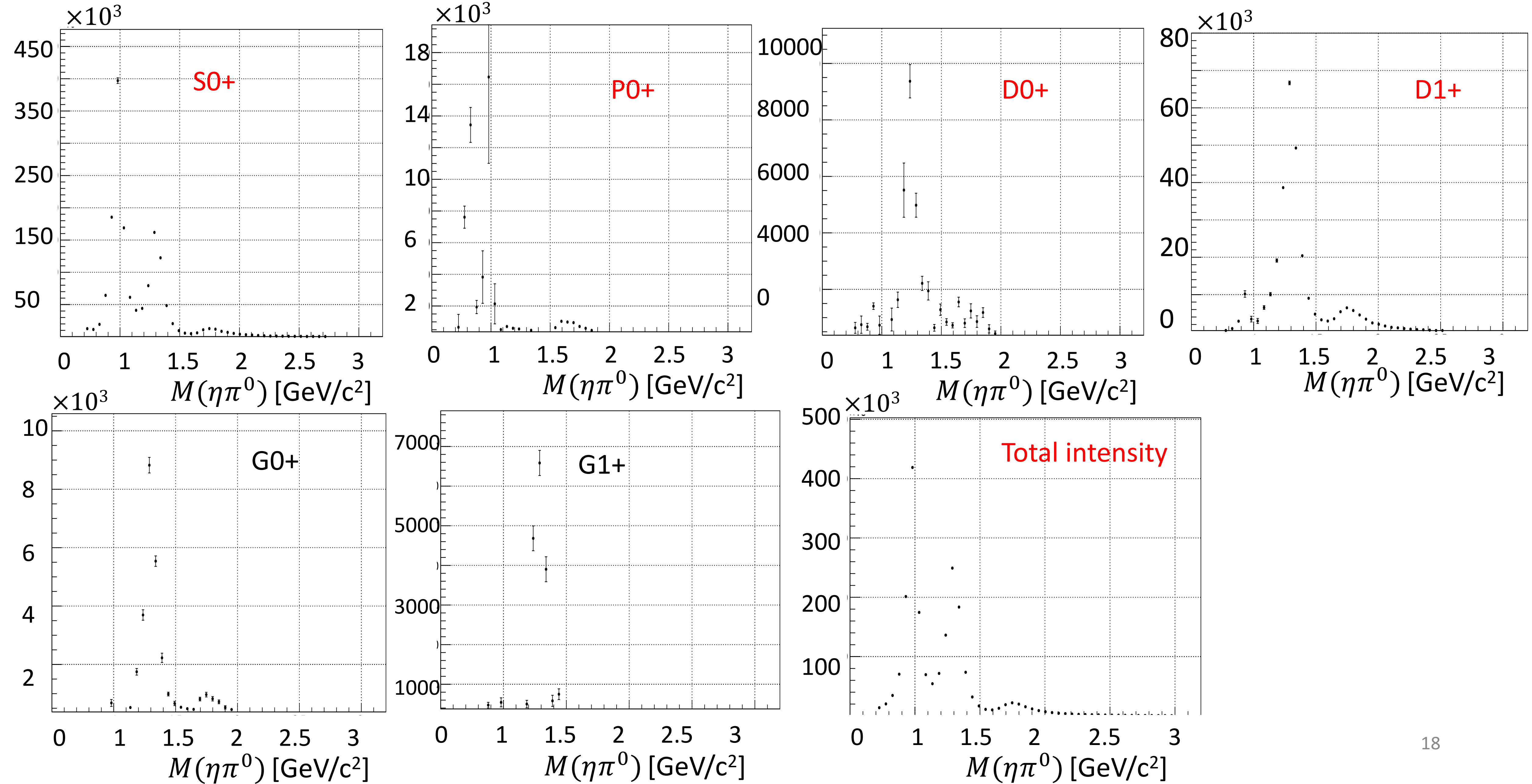
Fitting data with S0-, P0+, P1+, D0+, D1+, D2+ amplitude set

with S0-, P0+, D0+, D1+, G0+ ,G1+ .

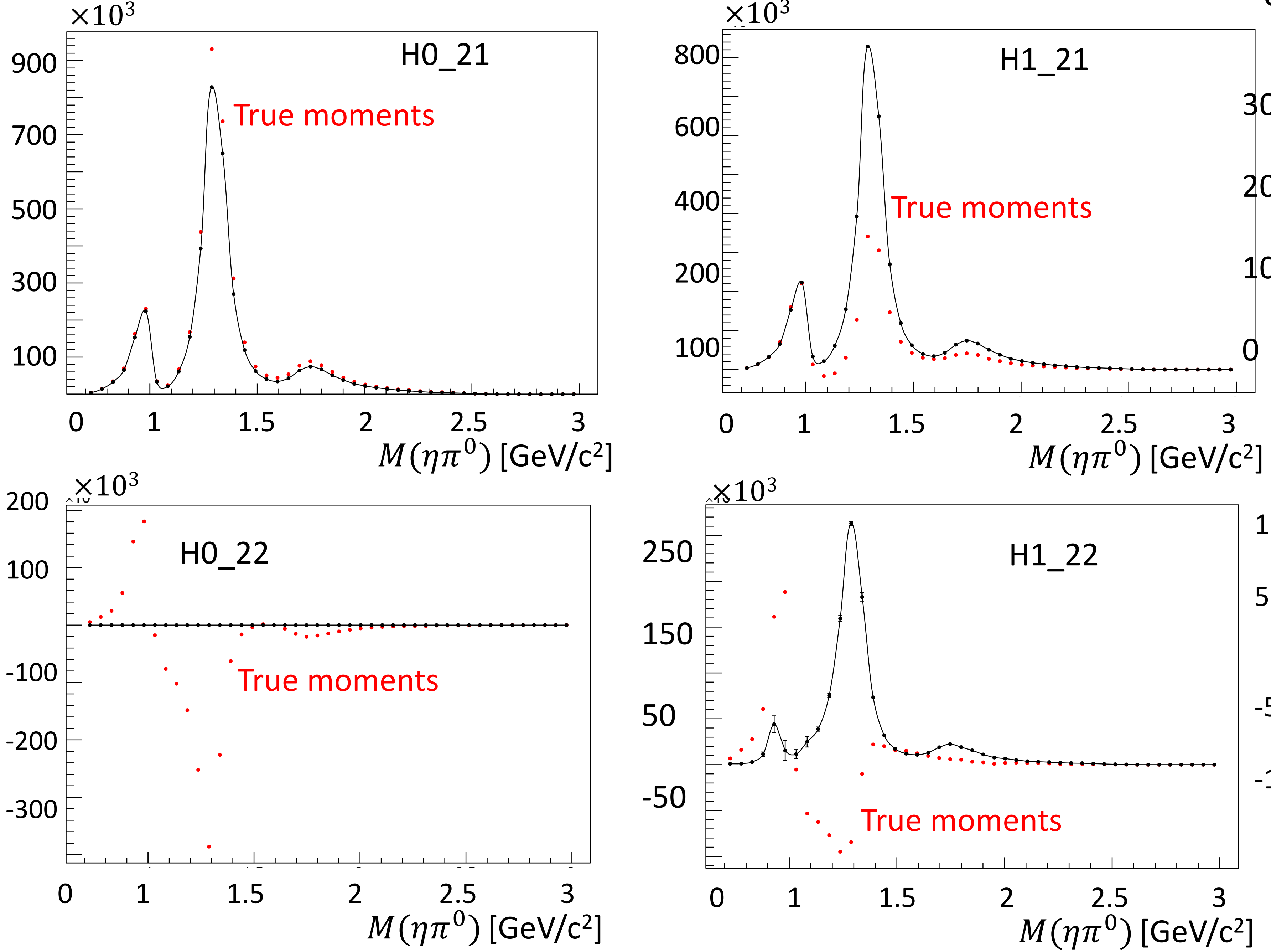
Fit 2 results (fitting in M and t bins)

Good starting values for fit parameters

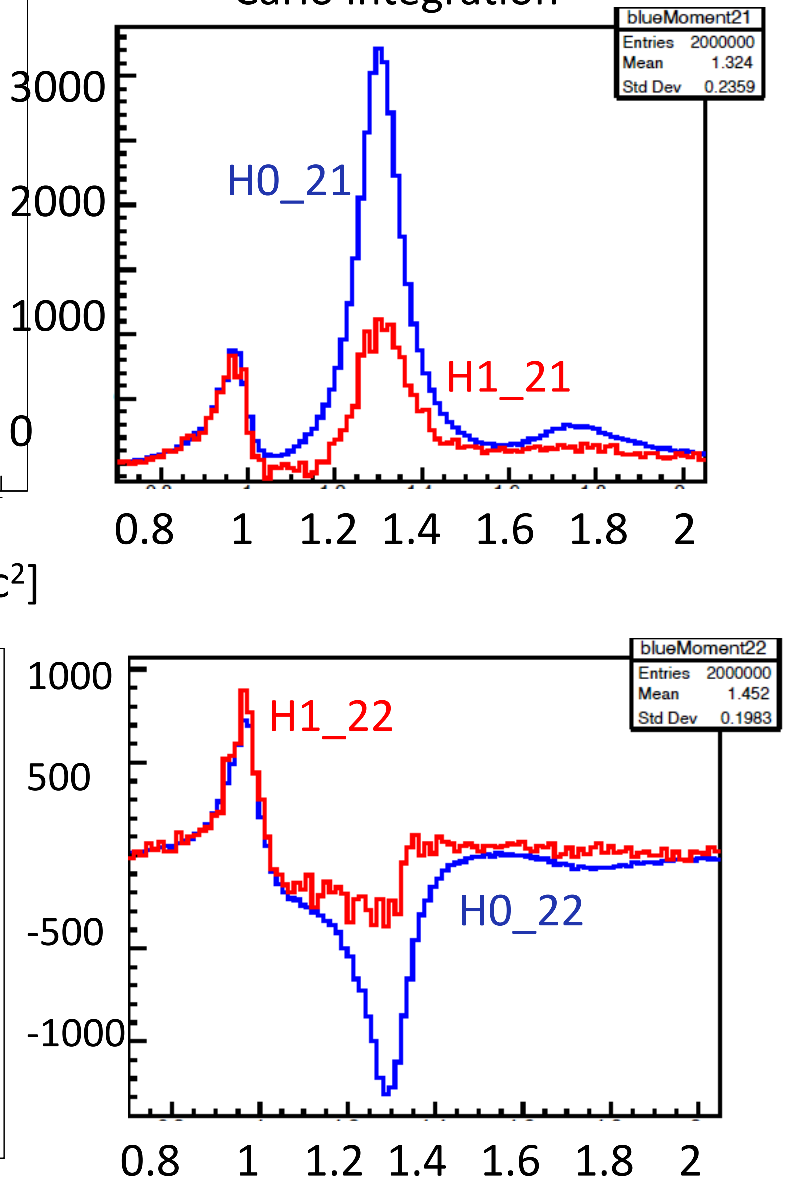


Polarized moments calculated with partial waves with uncertainties from bootstrapping

0<t<0.3 (GeV/c)^2 Calculated rom fitted amplitudes , with bootstrapping uncert.



Unnormalized moments from Monte Carlo integration

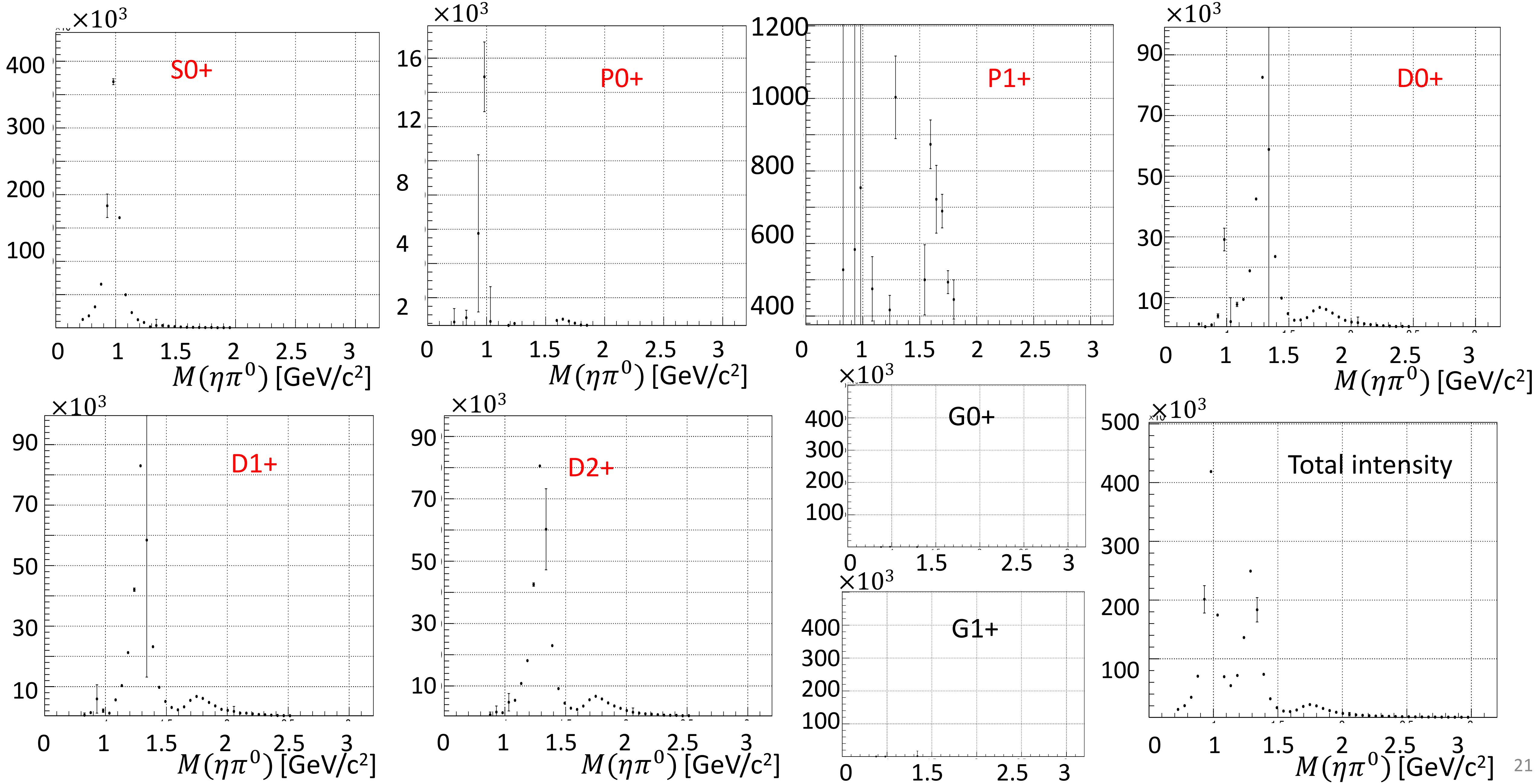


Fitting data with S0-, P0+, P1+, D0+, D1+, D2+ amplitude set

with S0-, P0+, P1+, D0+, D1+, D2+, G0+ ,G1+ .

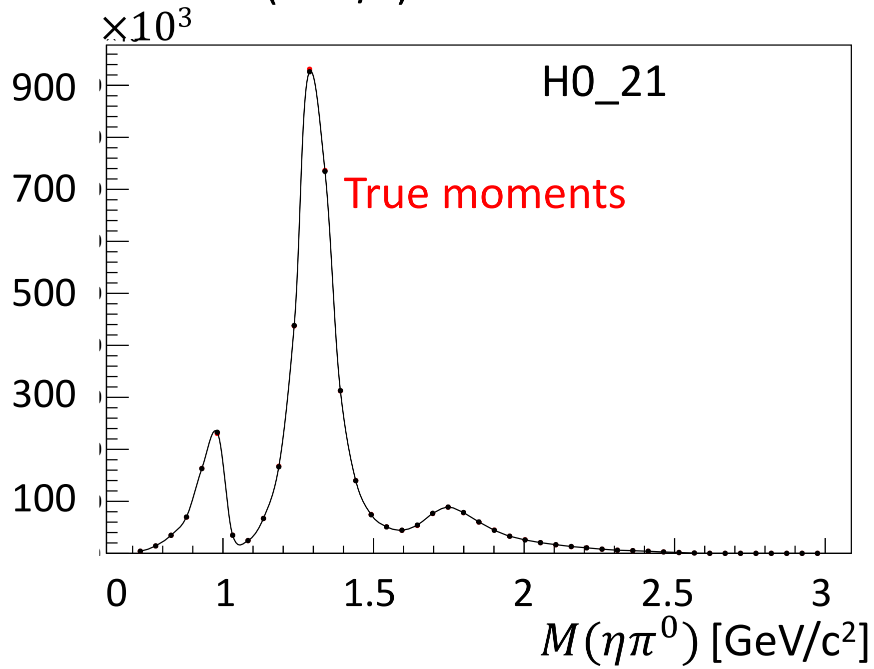
Fit 3 results (fitting in M and t bins)

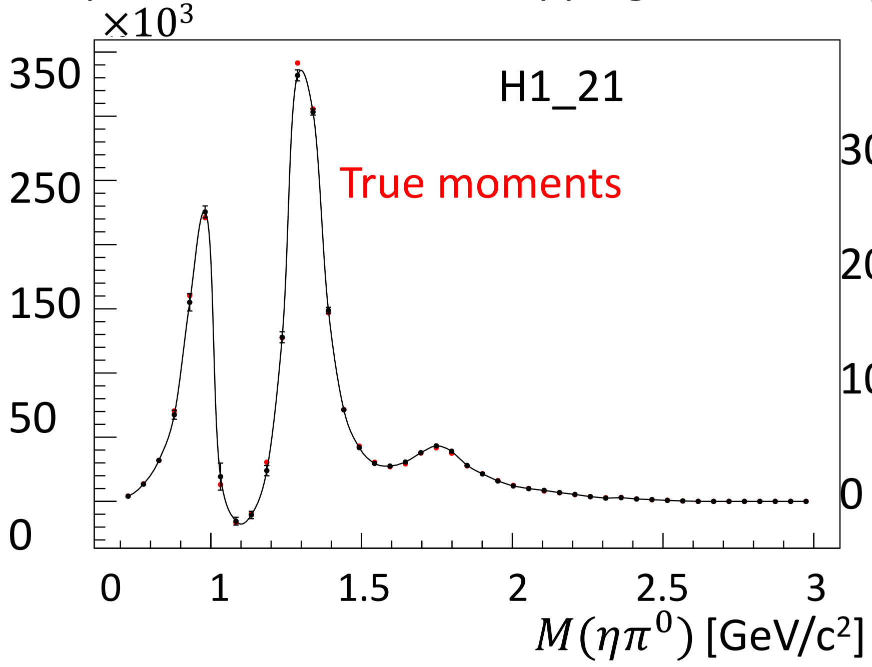
Good starting values for fit parameters



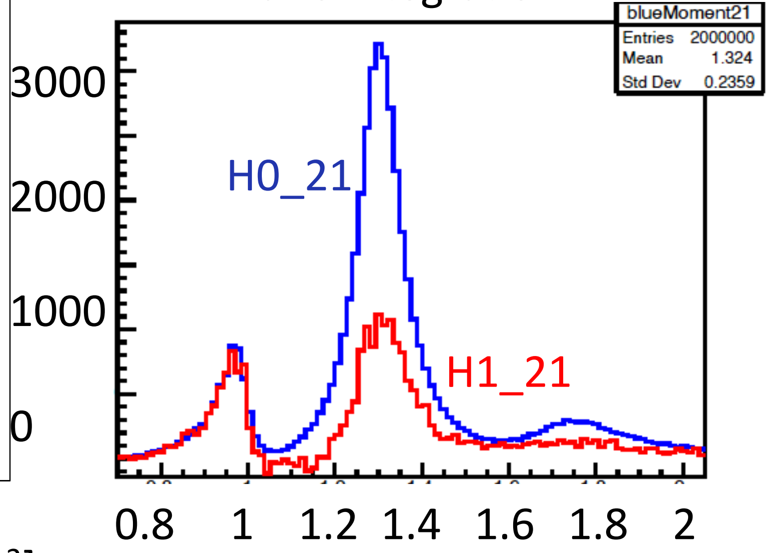
Polarized moments calculated with partial waves with uncertainties from bootstrapping

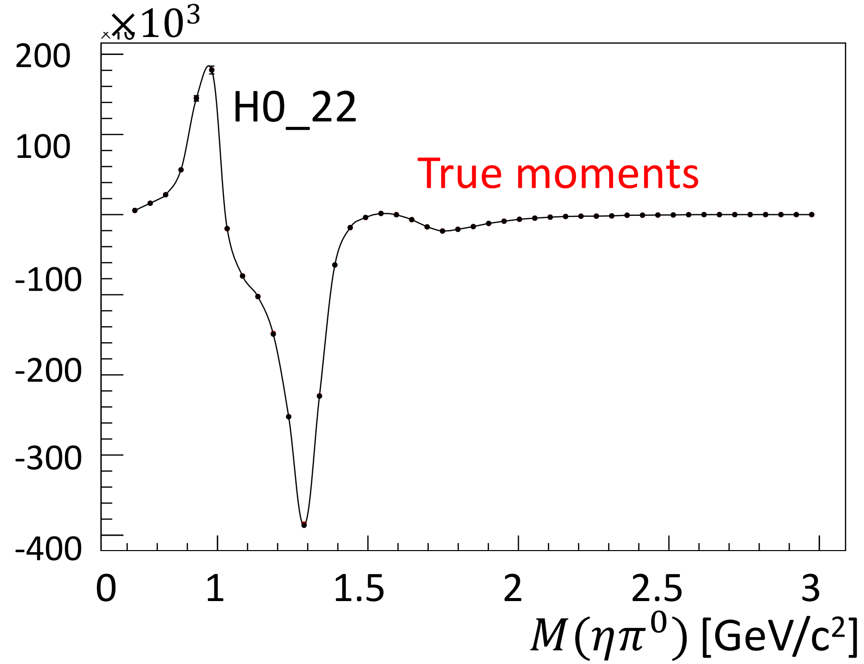
0<t<0.3 (GeV/c)^2 Calculated rom fitted amplitudes , with bootstrapping uncert.

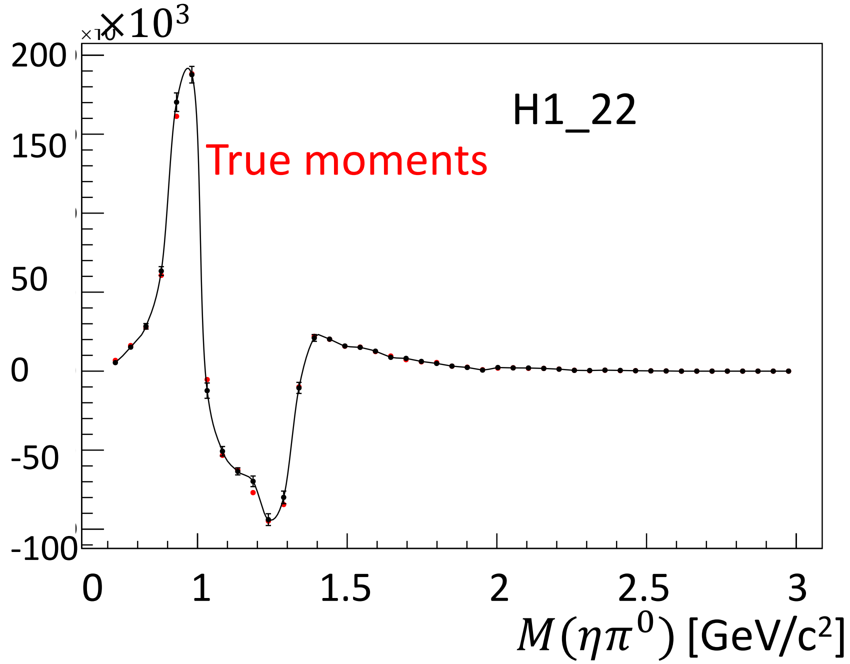


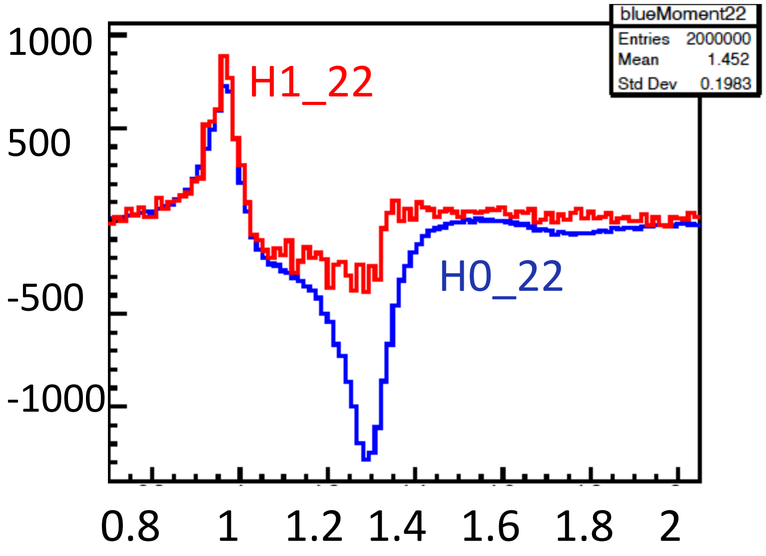


Unnormalized moments from Monte Carlo integration









Summary

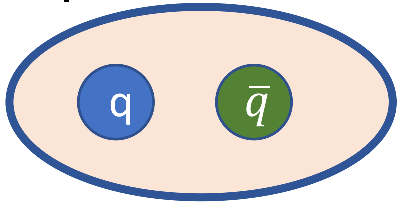
1. Have implemented and tested calculation of moments in terms of fitted partial waves using generated data sample.
2. Have also obtained unnormalized moments via Monte Carlo integration.
3. Moments obtained using both methods have similar shapes.
4. We show that leaving out any of the waves from original wave set results in poor extracted moments, while adding additional amplitudes leaves the result unchanged.
5. Future plans are

- Implement fitting of intensity to extract moments in AMPTOOLS.
- Search for exotic π_1 via amplitude analysis of GLUEX $\gamma p \rightarrow p\eta'\pi^0$data.

Backup slides

Mesons

Mesons in standard quark model



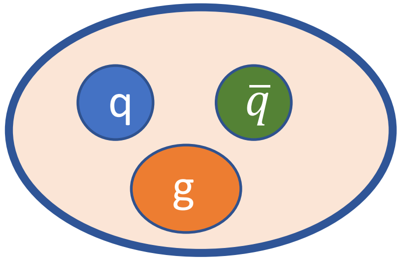
Classified as J^{PC} multilets:

$\vec{J} = \vec{L} + \vec{S}$,
$P = (-1)^{L+1} \rightarrow$ Spherical harmonics $(-1)^l$
$\times$ Product of individual parites of $q, \bar{q}$ (-1)
$C = (-1)^{L+S} \rightarrow$ Orbital angular momentum $(-1)^l$
$\times$ Flip of spin wavefunctions $(-1)^{S+1}$
$\times$ interchanging q and $\bar{q}$ (-1)

J- total angular momentum
S- total quark spin
L- orbital angular momentum between $q\bar{q}$ pair
P- parity
C- charge conjugation

J^{PC}=$\mathbf{0}^{--}$, $\mathbf{odd}^{-+}$ and $\mathbf{even}^{+-}$ **"exotic"** quantum numbers are not available.

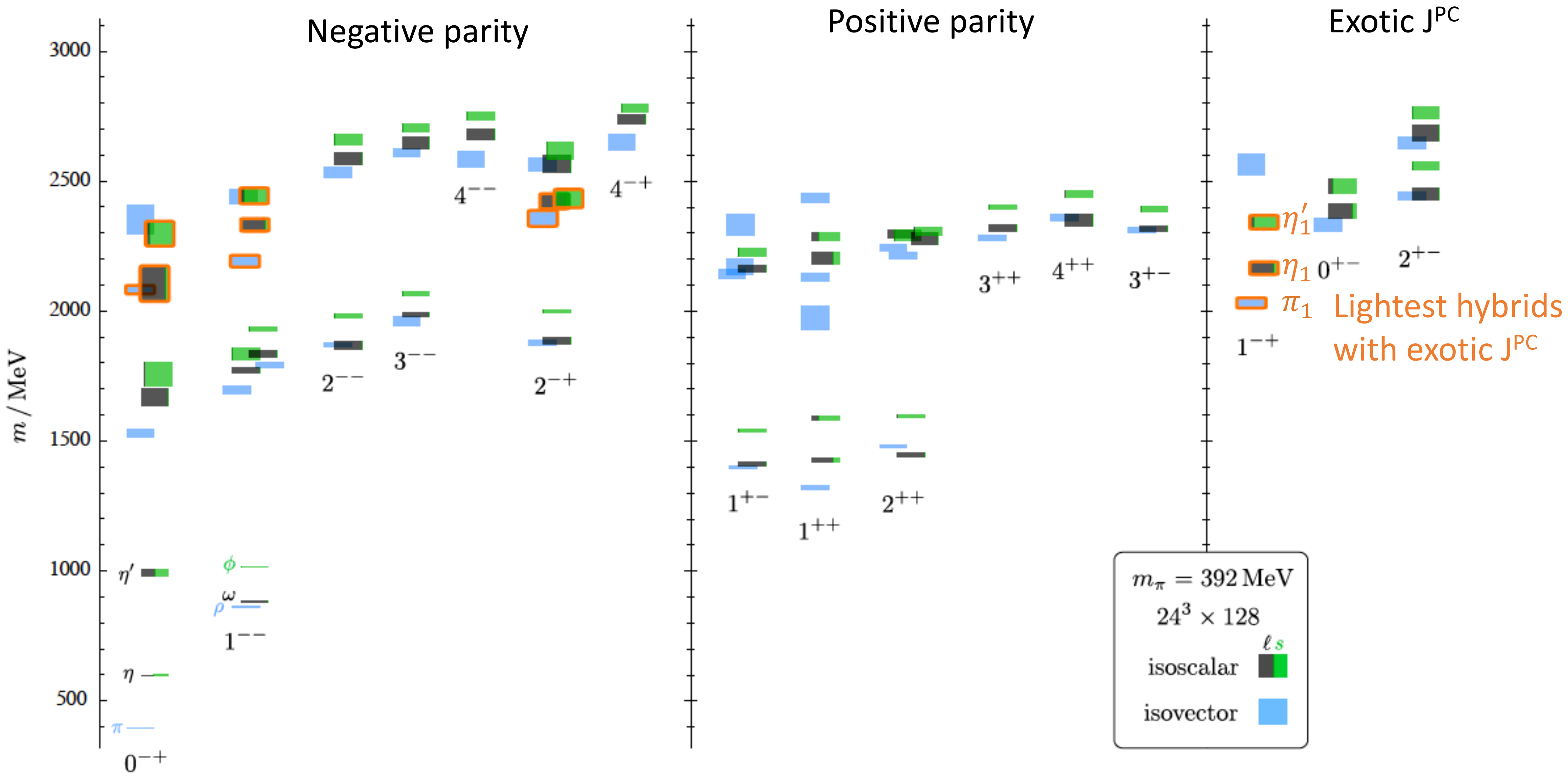
Hybrid mesons



Quark anti-quark pair coupled to valence gluon.
"Exotic" J^{PC} are also available.
Predicted by lattice QCD (quantum chromodynamics) calculations (Phys. Rev. D 88, 094505 (2013)).

Primary motivation of the GLUEX is the search for light hybrid mesons.

Isoscalar and Isovector hybrid spectrum from Lattice QCD



Phys. Rev. D 88, 094505 (2013)

Model for Intensity with polarized photon beam in $\eta^{(')}\pi^0$ photoproduction at GlueX

$$\vec{\gamma}(\lambda, p_\gamma)p(\lambda_1, p_N) \longrightarrow \pi^0(p_\pi)\,\eta(p_\eta)p(\lambda_2, p'_N)$$

$$I(\Omega,\Phi)=\frac{d\sigma}{dt\,dm_{\eta\pi}\,d\Omega\,d\Phi} = \kappa \sum_{\substack{\lambda,\lambda' \\ \lambda_1\lambda_2}} A_{\lambda;\lambda_1\lambda_2}(\Omega)\rho^\gamma_{\lambda\lambda'}A^*_{\lambda';\lambda_1\lambda_2}(\Omega)$$

$$I(\Omega,\Phi)= I^0(\Omega)\text{-}P_\gamma I^1(\Omega)\cos 2\Phi\text{-}\ P_\gamma I^2(\Omega)\sin 2\Phi$$

$$I^0(\Omega)=\frac{\kappa}{2}\sum_{\lambda,\lambda_1,\lambda_2} A_{\lambda;\lambda_1\lambda_2}(\Omega)\,A^*_{\lambda;\lambda_1\lambda_2}(\Omega),$$

$$I^1(\Omega)=\frac{\kappa}{2}\sum_{\lambda,\lambda_1,\lambda_2} A_{-\lambda;\lambda_1\lambda_2}(\Omega)\,A^*_{\lambda;\lambda_1\lambda_2}(\Omega),$$

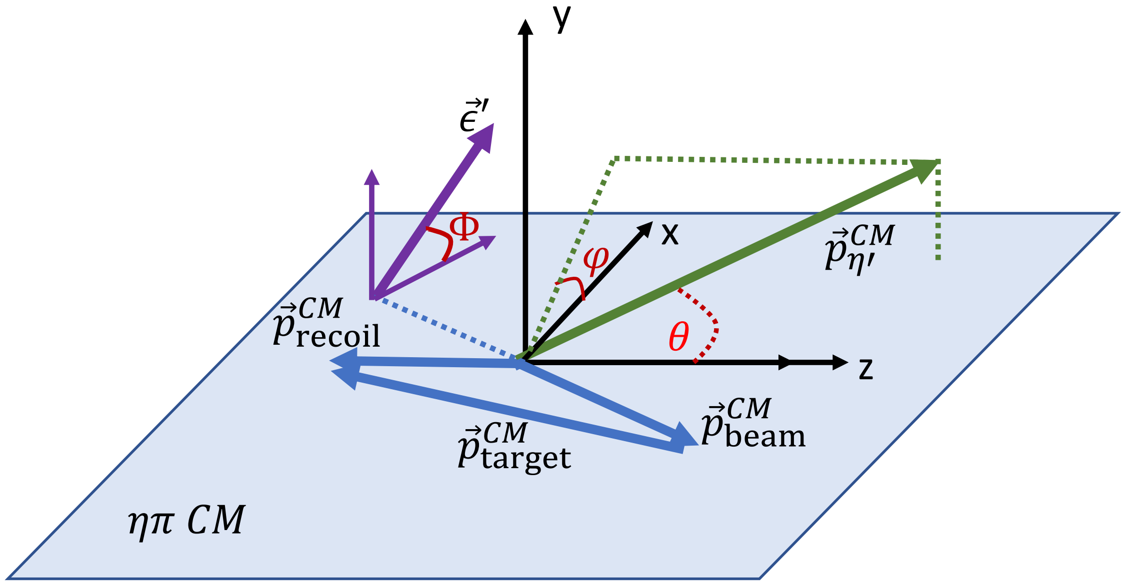
$$I^2(\Omega)=i\frac{\kappa}{2}\sum_{\lambda,\lambda_1,\lambda_2} \lambda A_{-\lambda;\lambda_1\lambda_2}(\Omega)\,A^*_{\lambda;\lambda_1\lambda_2}(\Omega),$$

Φ - angle between γ polarization vector $\vec{\epsilon}'$ and production plane
$\Omega = (\theta,\varphi)$- direction of η in helicity frame
P_γ- degree of linear polarization
λ -helicity
$A_{\lambda;\lambda_1\lambda_2}(\Omega)$-the reaction amplitude
$\rho^\gamma_{\lambda\lambda'}$-photon spin density matrix, encodes dependence on the polarization direction

$$\kappa = \frac{1}{(2\pi)^3}\frac{1}{4\pi}\frac{1}{2\pi}\frac{\lambda^{\frac{1}{2}}(m^2_{\eta\pi^0}, m^2_\pi, m^2_\eta)}{16 m_{\eta\pi^0}(s-m_N^2)^2}\frac{1}{2}$$

$\lambda(a,b,c) = a^2 + b^2 + c^2\text{-}2(ab+bc+ca)$
$s = (p_\gamma + p_N)^2$

Helicity frame



Partial wave amplitudes (production of the wave)

Decay amplitude into two pseudo-scalars (parity constraints, L cons.)

The partial wave amplitudes T^l are defined by: $A_{\lambda;\lambda_1\lambda_2}(\Omega) = \sum_{lm} T^l_{\lambda m;\lambda_1\lambda_2} Y^m_l(\Omega)$

We introduce reflectivity basis which allows to trade helicity λ for the reflectivity index $\epsilon=\pm1$, and express helicity amplitudes in terms of reflectivity amplitudes

$$T^l_{-1m;\lambda_1\lambda_2} = (-1)^m[{}^{(-)}T^l_{-m;\lambda_1\lambda_2} - {}^{(+)}T^l_{-m;\lambda_1\lambda_2}] \qquad T^l_{+1m;\lambda_1\lambda_2} = {}^{(-)}T^l_{m;\lambda_1\lambda_2} + {}^{(+)}T^l_{m;\lambda_1\lambda_2}$$

At high energies, t-channel exchange and natural (unnatural) exchanges contributes only to the $\epsilon=+(\epsilon=-)$ components in the reflectivity basis.

V. Mathieu et al. Phys. Rev. D 100, 054017 (2019)

Model for Intensity with polarized photon beam in $\eta^{(\prime)}\pi^0$ photoproduction at GlueX

Parity invariance implies

$${}^{(\epsilon)}T^{l}_{m;-\lambda_1-\lambda_2} = \epsilon(-1)^{\lambda_1-\lambda_2}\, {}^{(\epsilon)}T^{l}_{m;\lambda_1\lambda_2}$$

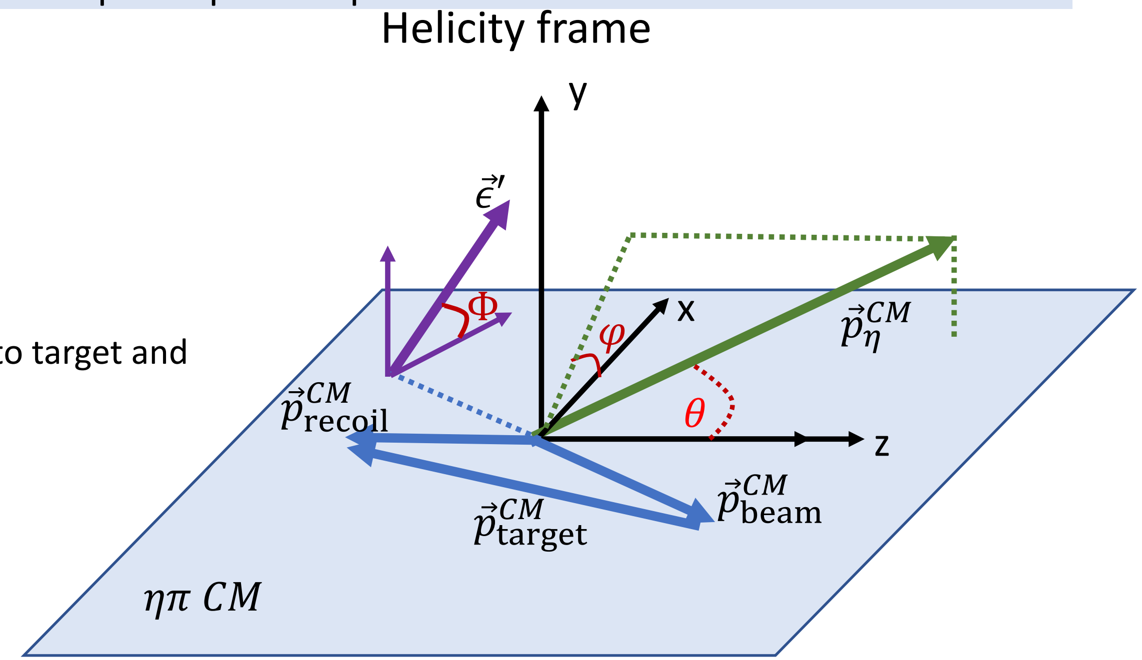
We take advantage of this constraint to define

$$l^{(\epsilon)}_{m;0}[l] = {}^{(\epsilon)}T^{l}_{m;++} \quad l^{(\epsilon)}_{m;1}[l] = {}^{(\epsilon)}T^{l}_{m;+-}$$

Are partial wave amplitudes for spin flip k=1 and spin non-flip k=0.
For each l , there are 2*2*(2l+1) complex partial waves with $\epsilon=\pm1$, k=0,1 corresponding to target and recoil helicities and m=-l,.....l.
There is no interference between $\epsilon=+$and $\epsilon=-$ intensities.
Define phase rotated spherical harmonics $Z^m_l(\Omega,\Phi) \equiv Y^m_l(\Omega)e^{-i\Phi}$

$$\mathrm{Re}Z^m_l(\Omega,\Phi)=\sqrt{\frac{2l+1}{4\pi}}\,d^l_{m0}(\theta)\cos(m\varphi-\Phi)$$

$$\mathrm{Im}Z^m_l(\Omega,\Phi)=\sqrt{\frac{2l+1}{4\pi}}\,d^l_{m0}(\theta)\sin(m\varphi-\Phi)$$



Intensity that involves four coherent sums for each configuration of nucleon spin:

$$I(\Omega,\Phi) = 2\kappa\sum_k\left\{(1-P_\gamma)\left|\sum_{l,m}[l]^{(-)}_{m;k}\mathrm{Re}[Z^m_l(\Omega,\Phi)]\right|^2 + (1-P_\gamma)\left|\sum_{l,m}[l]^{(+)}_{m;k}\mathrm{Im}[Z^m_l(\Omega,\Phi)]\right|^2 + (1+P_\gamma)\left|\sum_{l,m}[l]^{(+)}_{m;k}\mathrm{Re}[Z^m_l(\Omega,\Phi)]\right|^2 + (1+P_\gamma)\left|\sum_{l,m}[l]^{(-)}_{m;k}\mathrm{Im}[Z^m_l(\Omega,\Phi)]\right|^2\right\}$$

GlueX doc-4094 (M. Shepherd)

Helicity-non-flip amplitudes dominate and we set the helicity-flip amplitudes to zero. This is not restrictive as the target is not polarized in GlueX, and the measured intensities are not sensitive to the details of the nucleon helicity structure.
Natural parity exchanges (corresponding to the amplitudes with ϵ=+1) dominate in the energy range of interest.

- Bin data in small bins of $m_{\eta\pi}$, t and E_γ with constant $[l]^{(-)}_{m;k}$
- Fit data using extended unbinned (in (θ,φ)) maximum likelihood method

$$\ln L(l) = \sum_{i=1}^{N}\ln I(l,\theta,\varphi) - \int I(l,\theta,\varphi)\eta(\theta,\varphi)\,d\Omega$$

$\eta(\theta,\varphi)$ -acceptance

- Minimize –lnL using MINUIT, to find V

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Plotting $cos\theta$ distributions of different waves from fit results

Mass dependent fit

1. Assume acceptance $\eta(\theta, \varphi) = 1$ and use same MC sample for both accepted and generated MC.
2. Execute fitting:

fit -c solution.cfg

3. To plot the results in GUI:

twopi_plotter_mom etapi0.fit -g

Model predicted number of observed events is calculated using MC integration:

$$\mu = \int I(\theta, \varphi)\eta(\theta, \varphi)\, d\Omega \approx \frac{4\pi}{N_{Gen}} \sum_{N_{Acc}} I(\theta, \varphi)$$

If we weight each MC event with following weight, we will obtain fit results to be compared to data:

$$\omega_i = \frac{4\pi}{N_{MC}} I(\theta_i, \varphi_i)$$

Partial waves $[l]^{(+)}_{m;k}$

Consider positive reflectivity and nucleon helicity non-flip
The contribution of resonance R to the wave l reads:

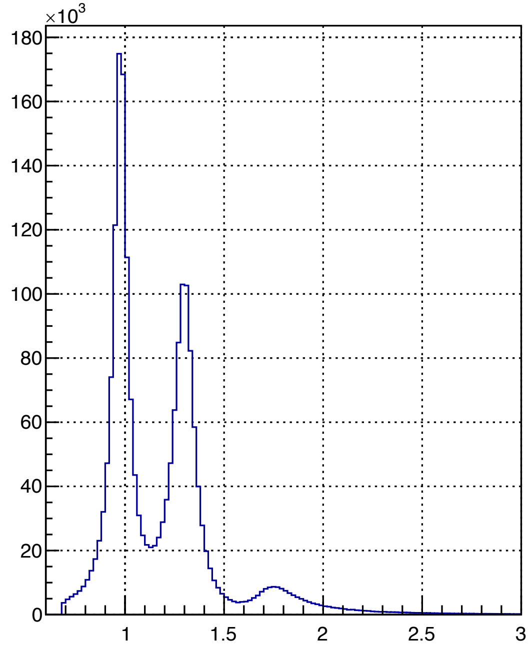
$[l]^{(+)}_{m;0} = N_R F \Delta_R$

$\Delta_R = \frac{\sqrt{m_R \Gamma_R}}{\pi(m_R{}^2 - m_{\eta\pi}{}^2 - i m_R \Gamma)}$

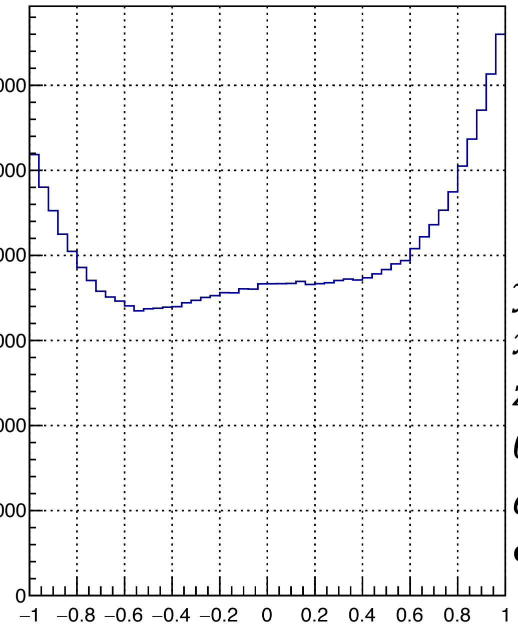
$\Gamma = \Gamma_R \left(\frac{m_R}{m_{\eta\pi}}\right)\left(\frac{q}{q_0}\right)\left(\frac{F^2}{F_0{}^2}\right)$

F, F_0- Blatt-Weisskopf barrier factors used in Amptools
q, q_0- Breakup momenta from AmpTools

Generated $2*10^6$ $(p\eta'\pi^0)$ events with AmpTools

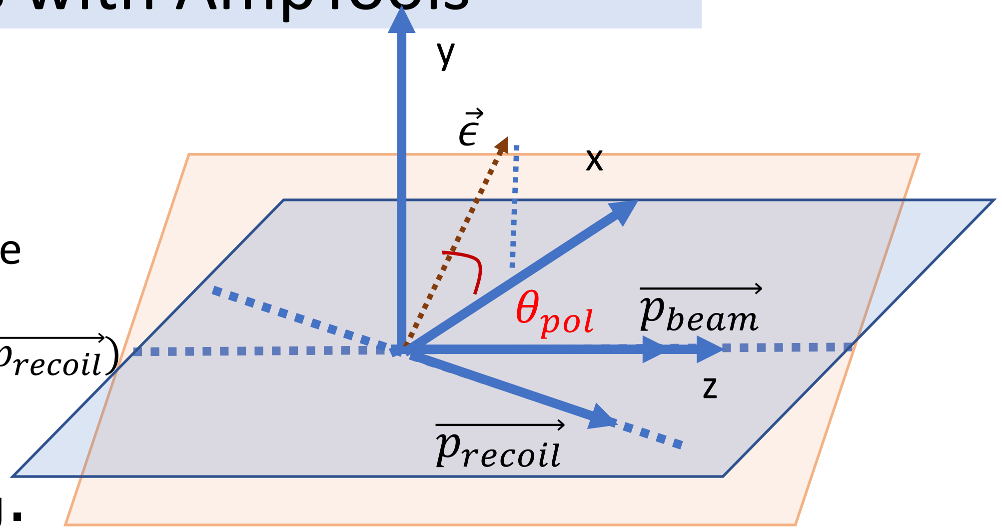


$M(\eta'\pi^0)$ [GeV/c²]



$\cos\theta_{GJ}$

Lab frame



$\vec{y} = \overrightarrow{p_{beam}} \times (-\overrightarrow{p_{recoil}})$

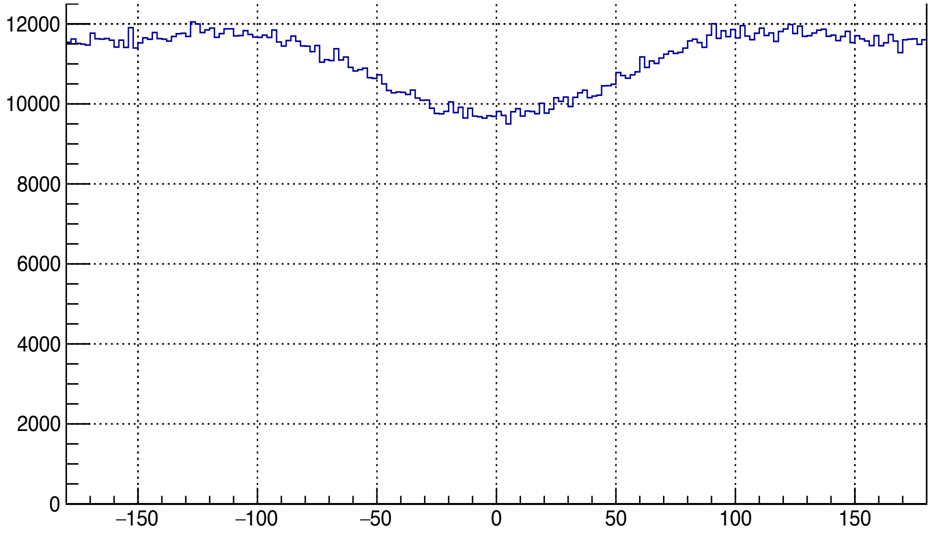
$\vec{x} = \vec{y} \times \overrightarrow{p_{beam}}$

$\vec{z} = \vec{x} \times \vec{y}$

θ_{pol}=1.7 Deg.

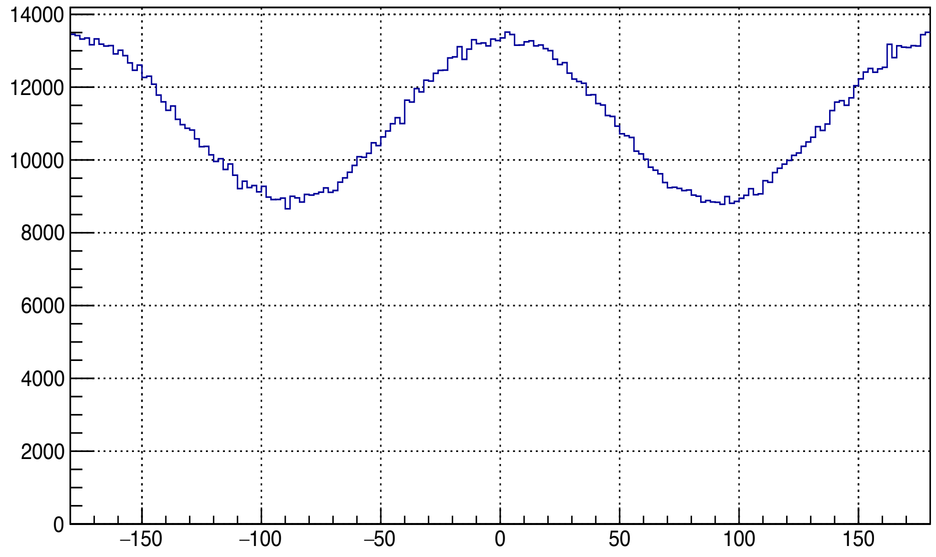
$\vec{\epsilon} = (\cos(\theta_{pol}), \sin(\theta_{pol}), 0)$

$\Phi = \mathrm{arctg}(\vec{y} \cdot \vec{\epsilon}, \overrightarrow{p_{beam}} \cdot (\vec{\epsilon} \times \vec{y}))$



φ

Lab frame



Φ

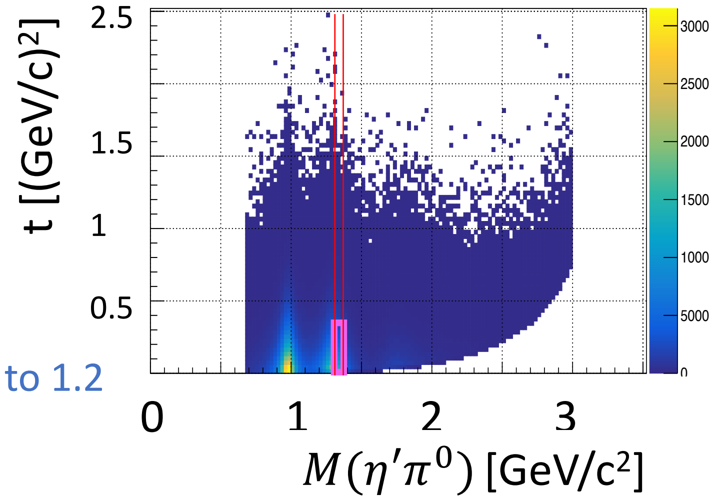
Results for bin M=1.37 and t<0.3

Amplitudes used in fitting are S0+, P0+, P1+, D0+, D1+, D2+. Good starting values for fit parameters

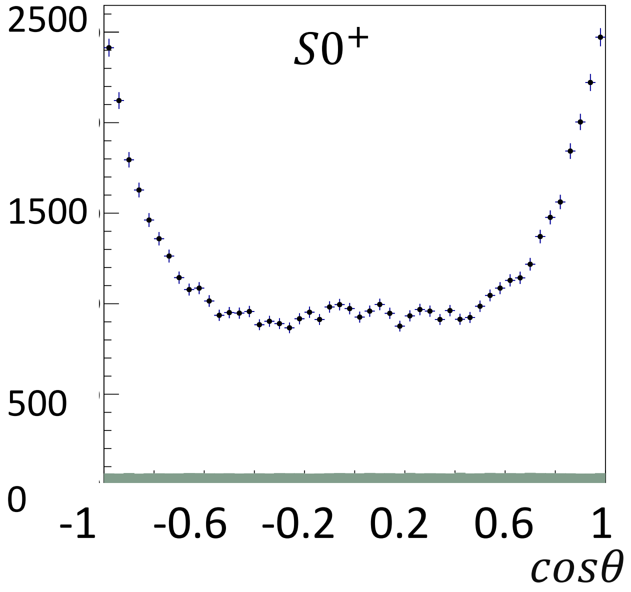
Bin M , t

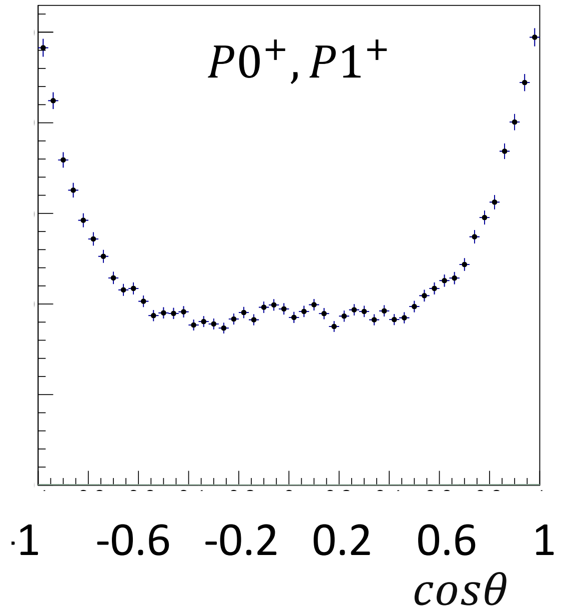
$M(\eta\pi^0)$ range from 0.7 to 3
N bins=45
Bin width≈ 0.051 GeV/c²

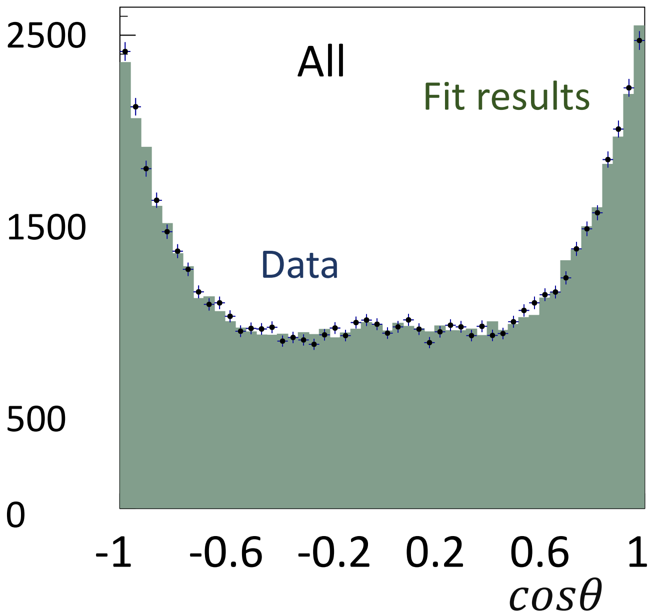
t range from 0 to 1.2
N bins=4
Bin width≈ 0.3 (GeV/c)2

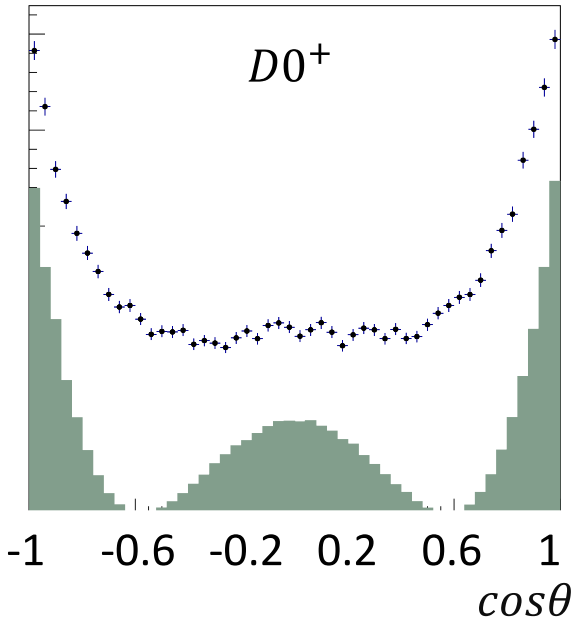


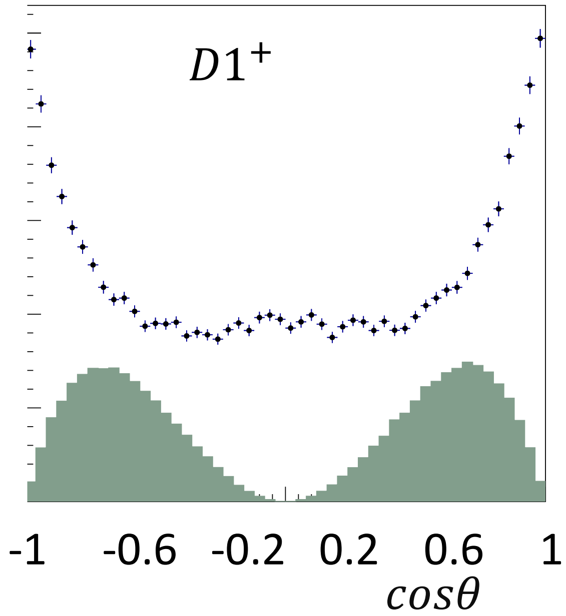
Fit results

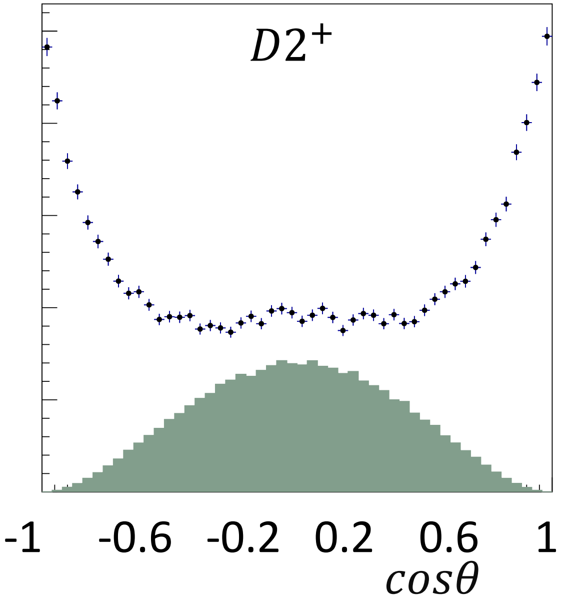








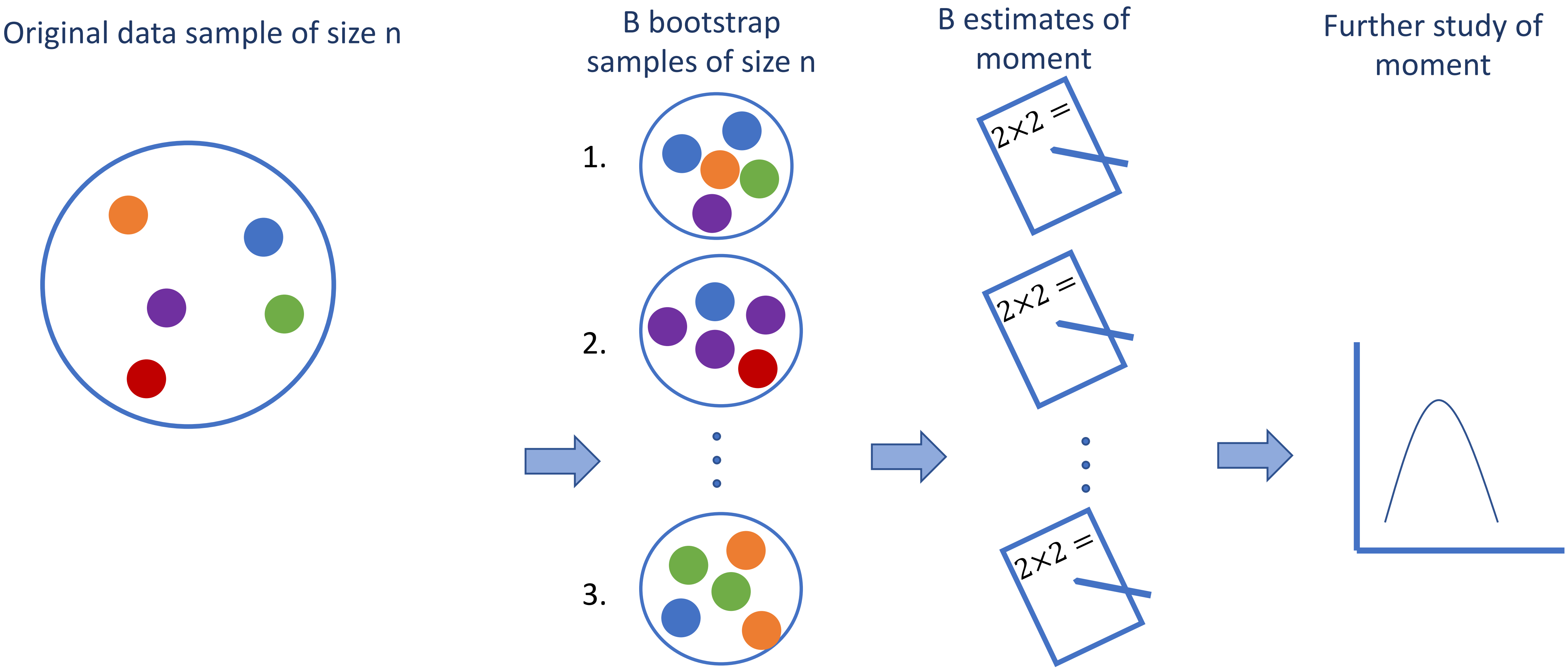




Fitting data with S0-, P0+, P1+, D0+, D1+, D2+ amplitude set

with S0-, P0+, D0+, D1+, G0+ ,G1+ .

Bootstrapping method for estimation of uncertainties

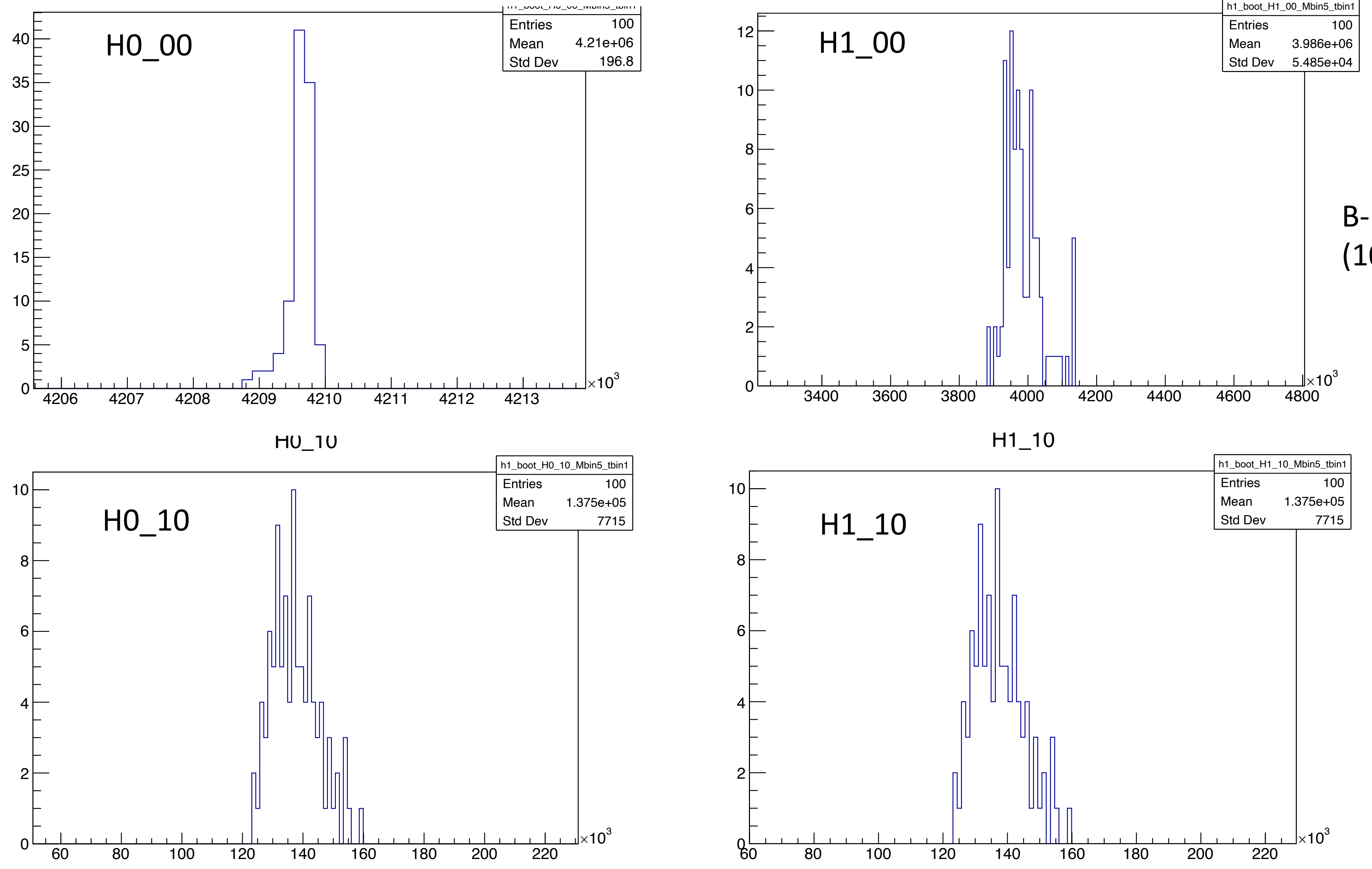


1. Draw a Bootstrap Sample from the original sample data with replacement with size n.
2. Evaluate intensity for each Bootstrap Sample which will result in B estimates of intensity.
3. Construct a histogram of B estimates of intensity and use it to make further statistical inference, such as:

- Estimating the standard error of statistic for Intensity.

Distributions of moment values from 100 bootstrapping samples for **M bin=5 and t bin=1**

M~0.93 GeV/c^2 $0<t<0.3$ (GeV/c)^2



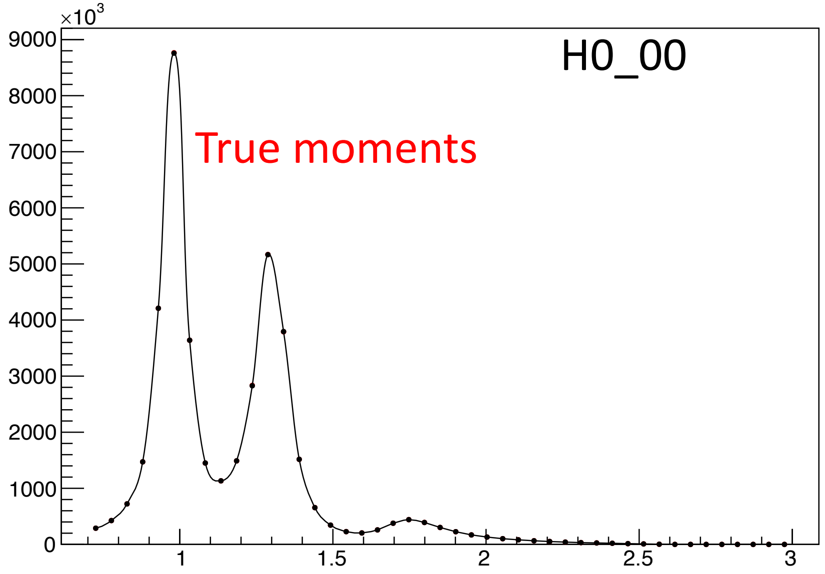
$$\sigma = \sqrt{\frac{\sum_i^B (I_i - I_{mean})^2}{B}}$$

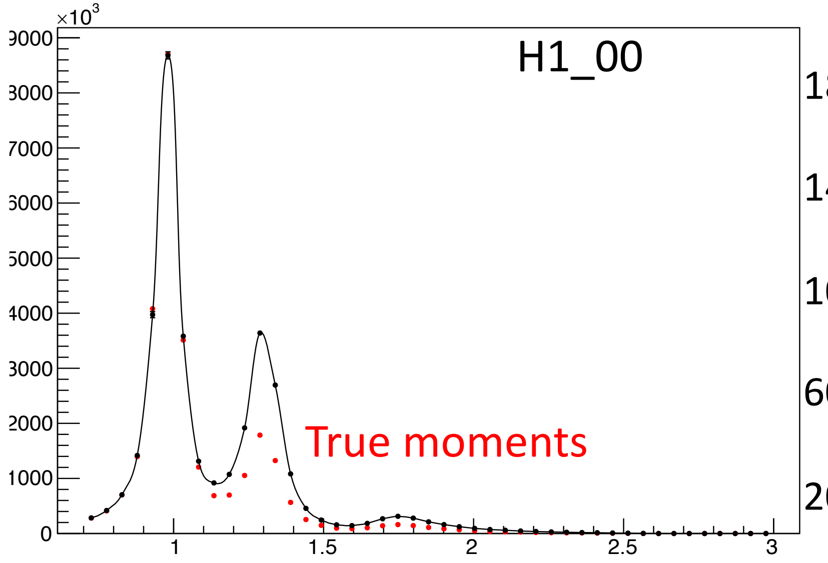
B- number of Bootstraps (100 in this case)

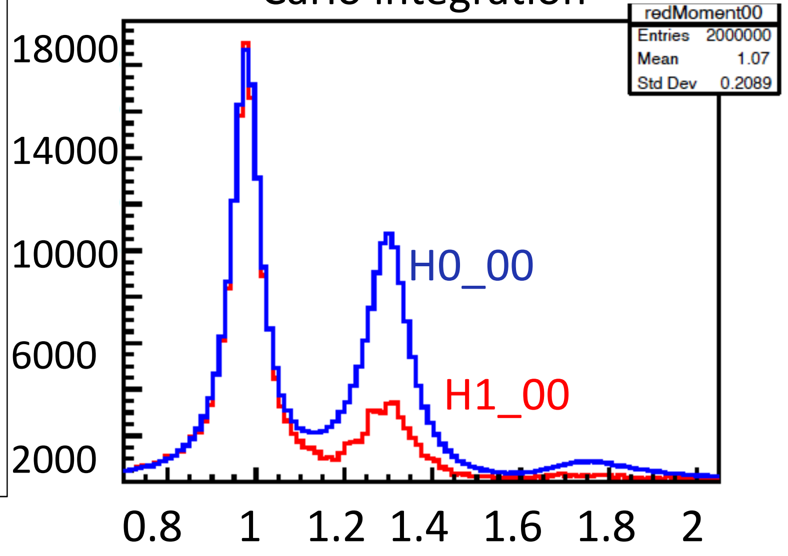
Polarized moments calculated with partial waves with uncertainties from bootstrapping

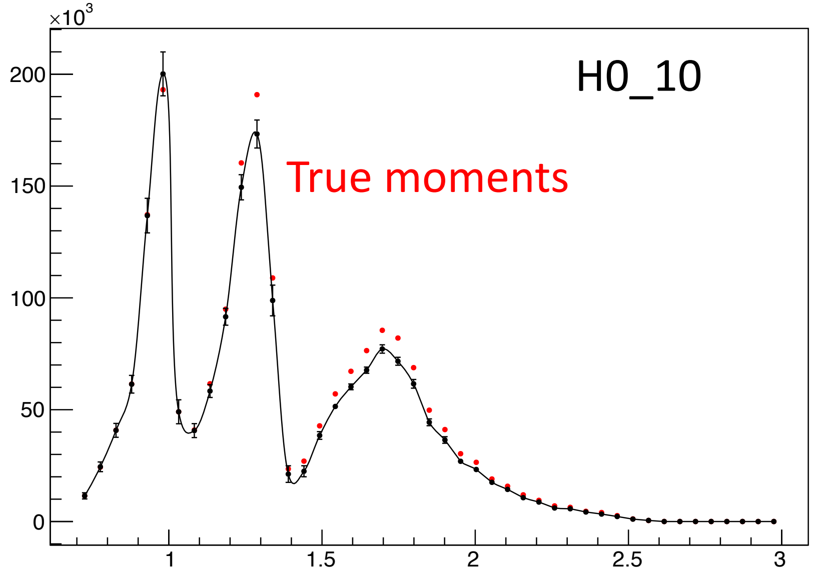
0<t<0.3 $(GeV/c)^2$

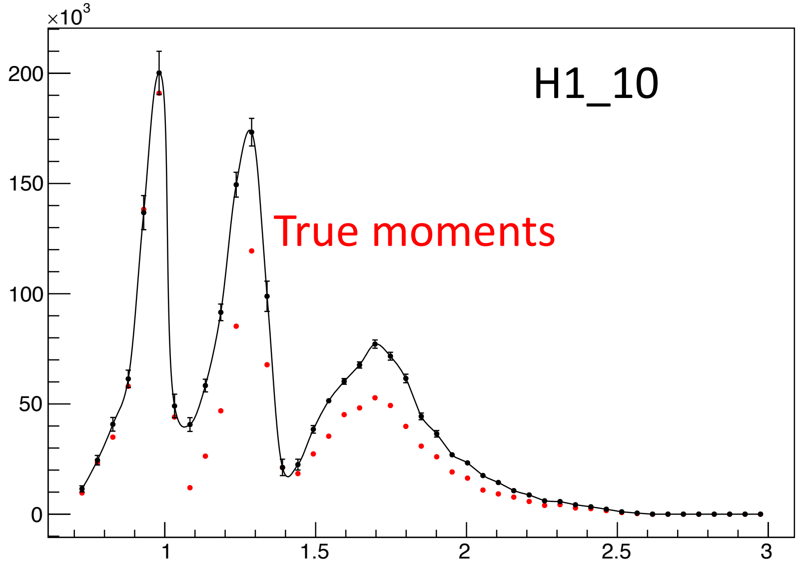
Calculated from fitted amplitudes, with bootstrapping uncert. Unnormalized moments from Monte Carlo integration

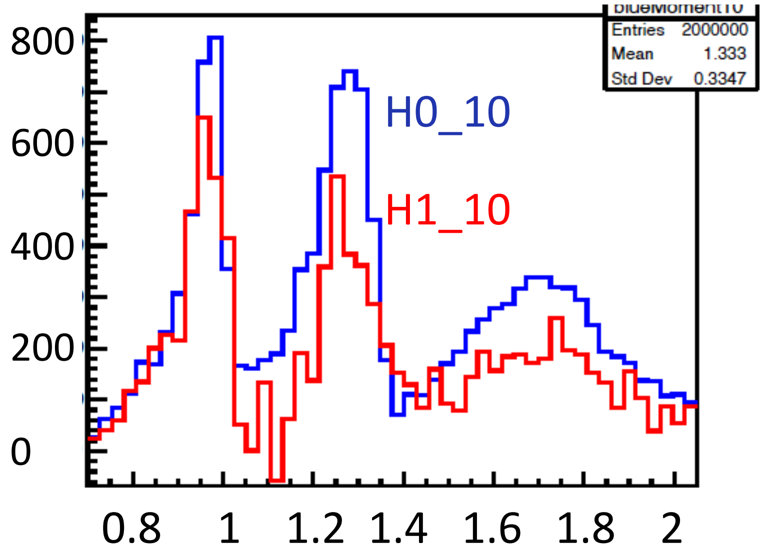








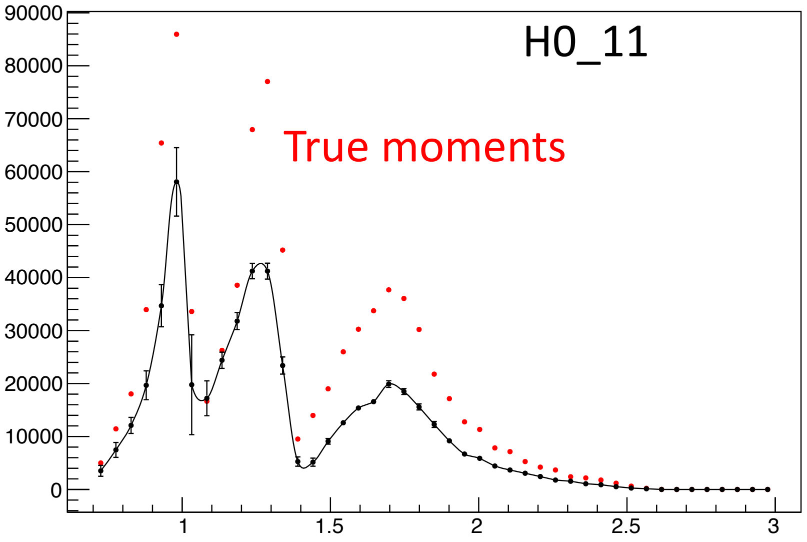


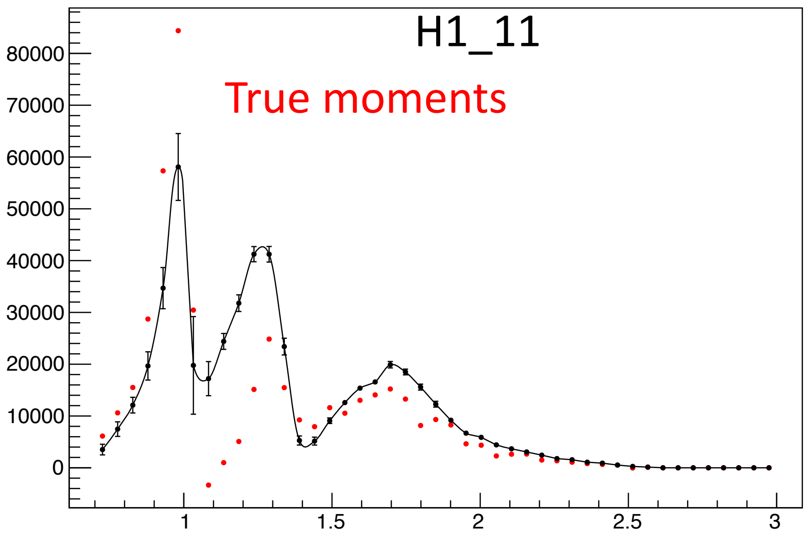


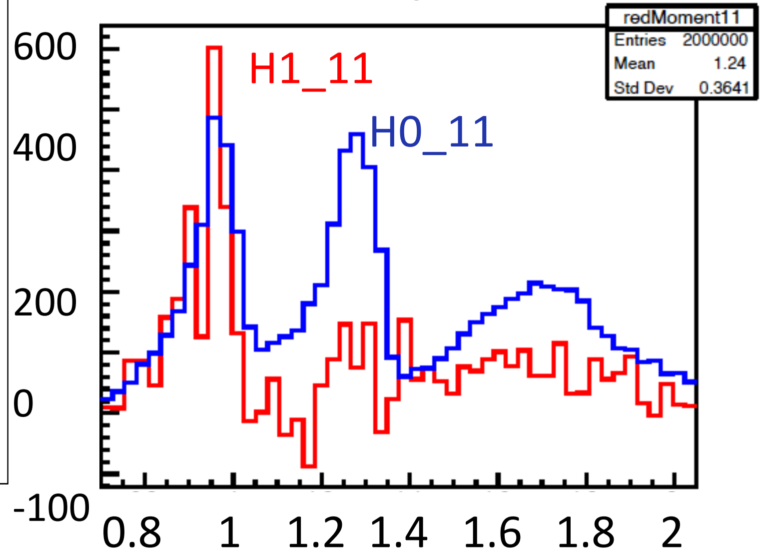
Polarized moments calculated with partial waves with uncertainties from bootstrapping

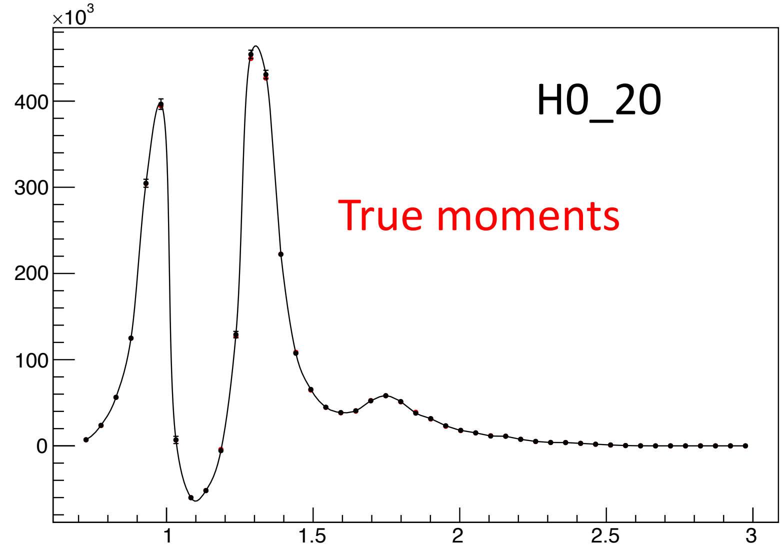
$0<t<0.3$ $(GeV/c)^2$ Calculated from fitted amplitudes , with bootstrapping uncert.

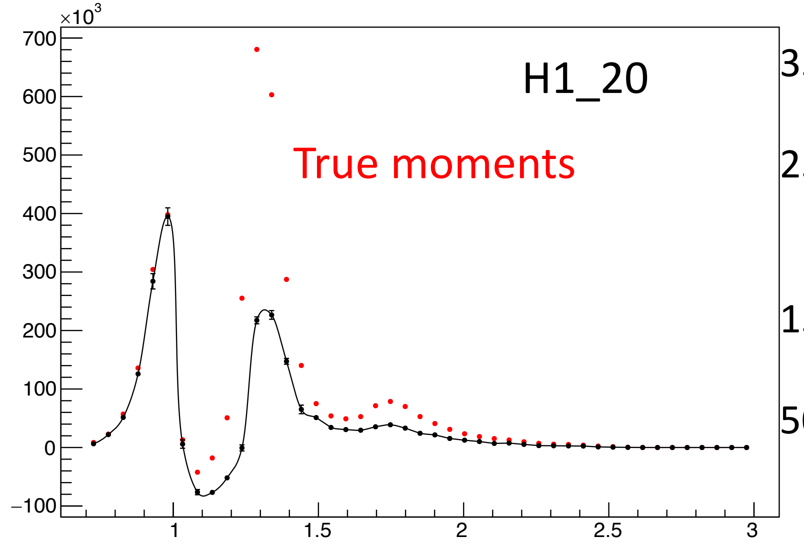
Unnormalized moments from Monte Carlo integration

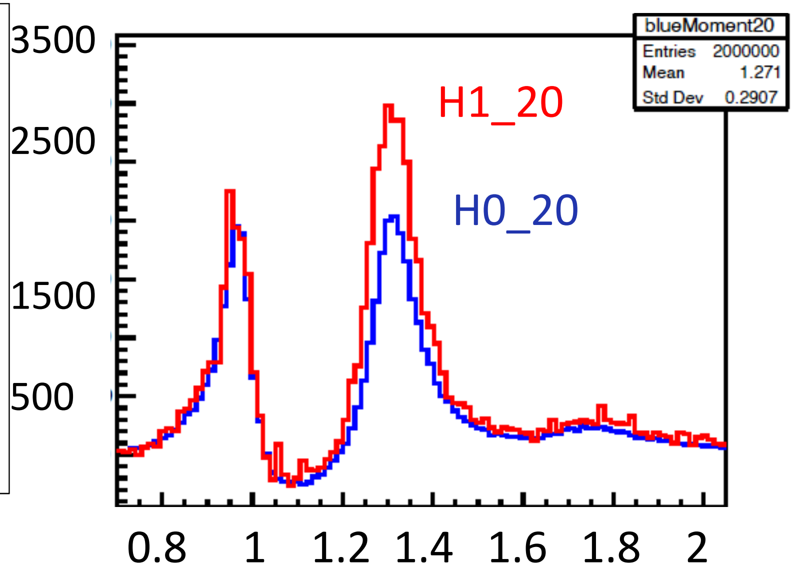










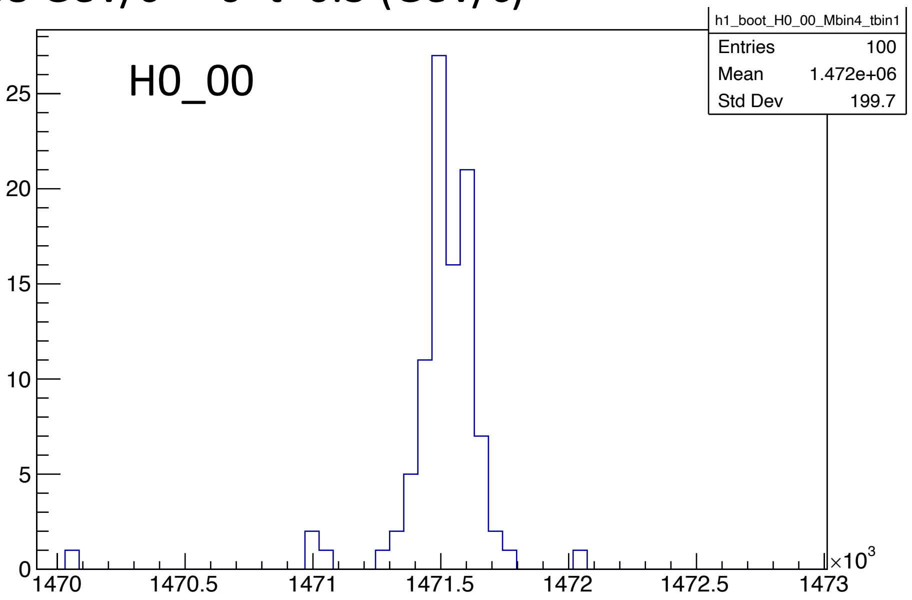


Fitting data with S0-, P0+, P1+, D0+, D1+, D2+ amplitude set

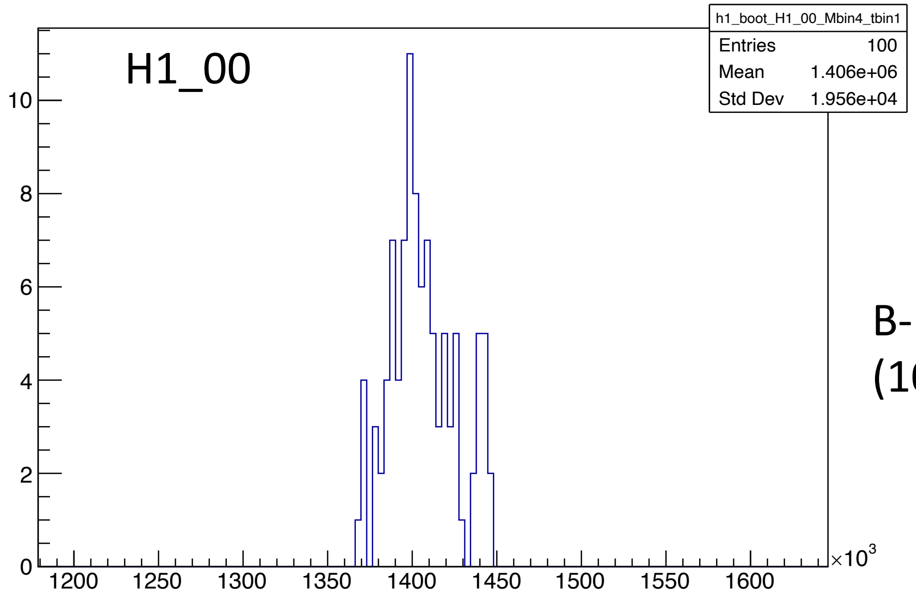
with S0-, P0+, P1+, D0+, D1+, D2+, G0+ ,G1+ .

Distributions of moment values from 100 bootstrapping samples for **M bin=4 and t bin=1**

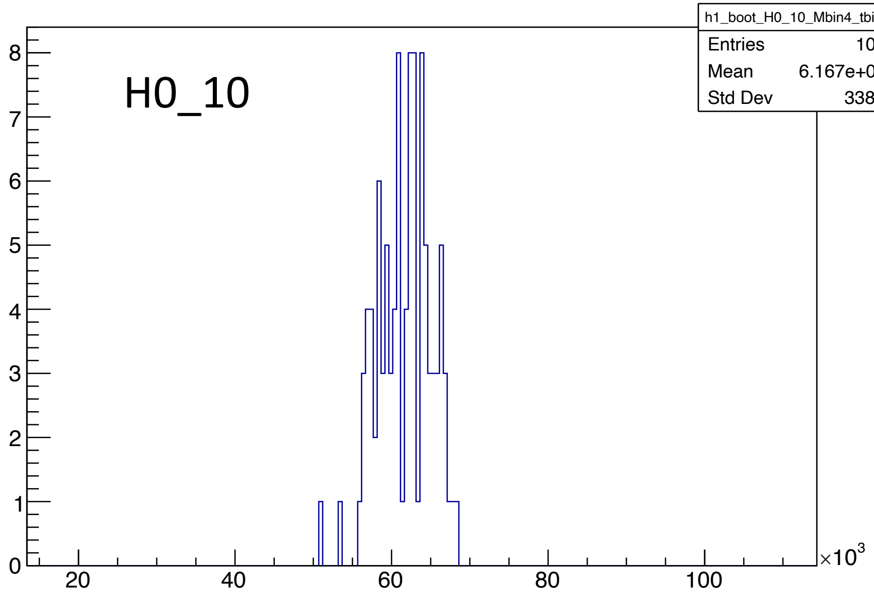
M~0.88 GeV/c^2 0<t<0.3 (GeV/c)^2



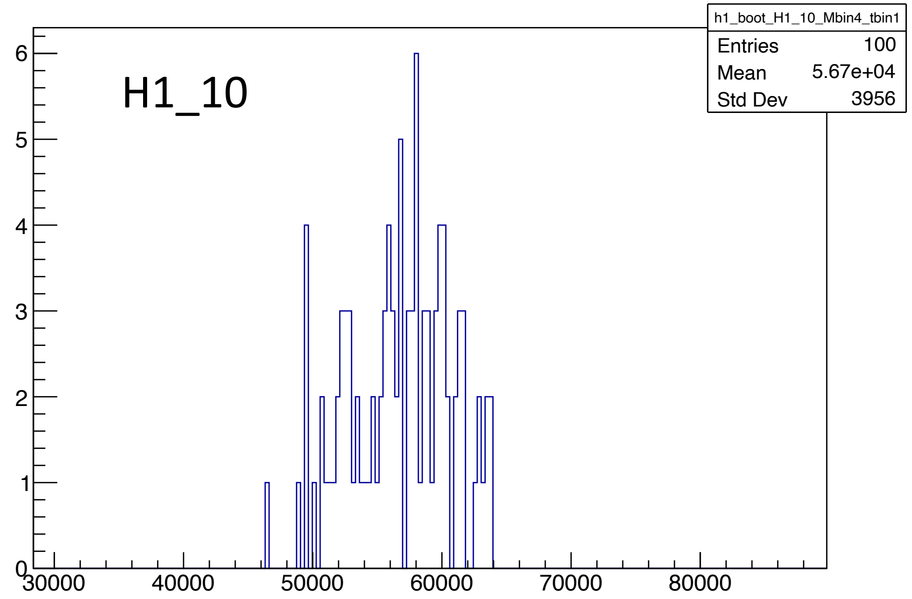
H1_00



H0_10



H1_10



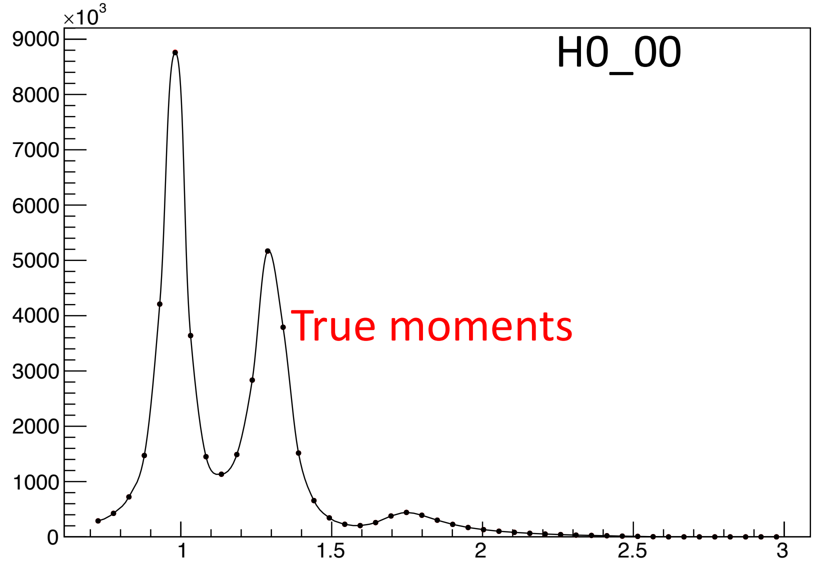
$$\sigma = \sqrt{\frac{\sum_i^B (I_i - I_{mean})^2}{B}}$$

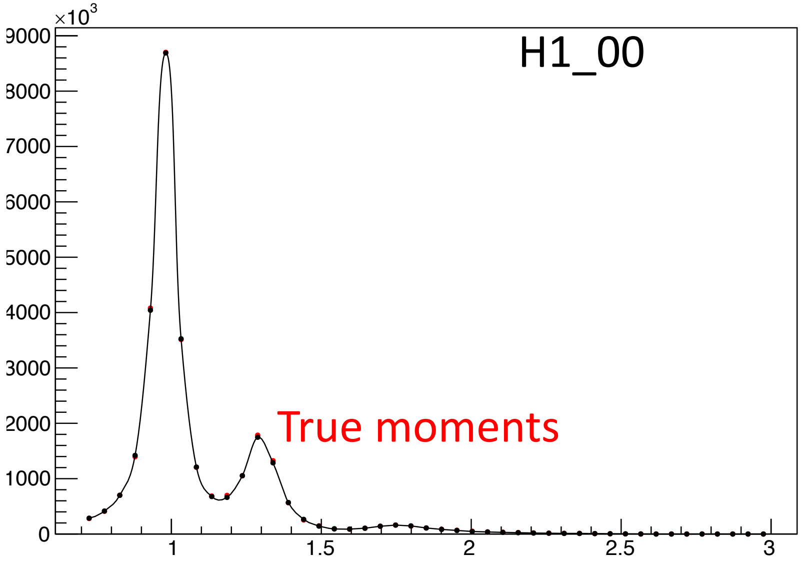
B- number of Bootstraps (100 in this case)

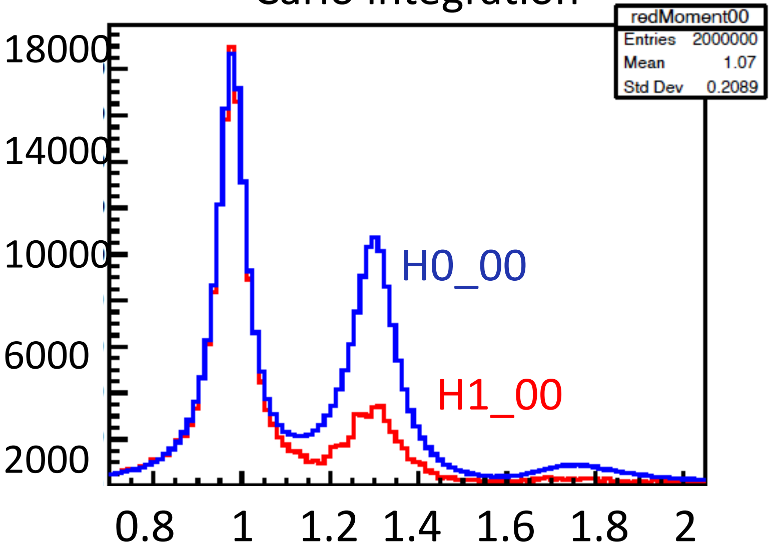
Polarized moments calculated with partial waves with uncertainties from bootstrapping

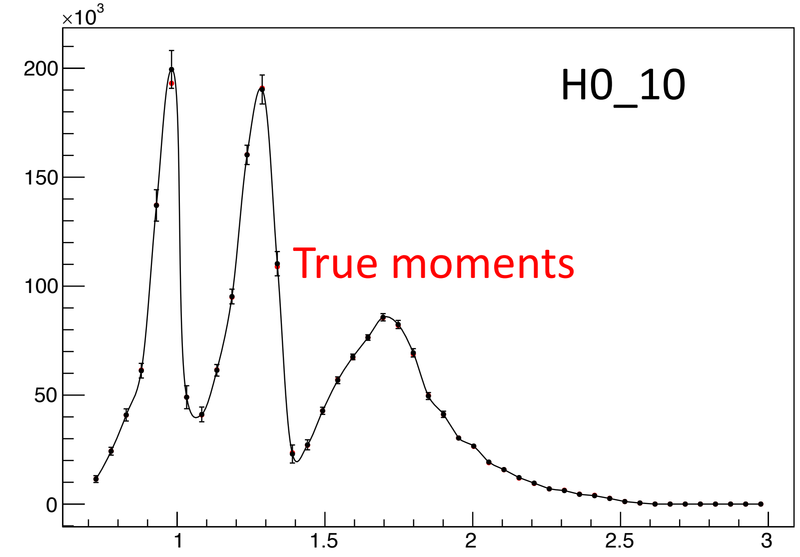
0<t<0.3 (GeV/c)^2

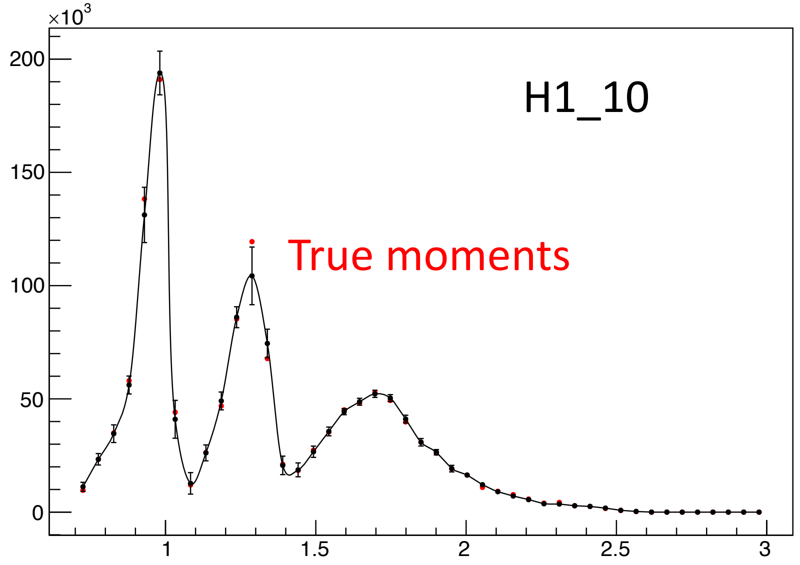
Calculated from fitted amplitudes, with bootstrapping uncert. Unnormalized moments from Monte Carlo integration

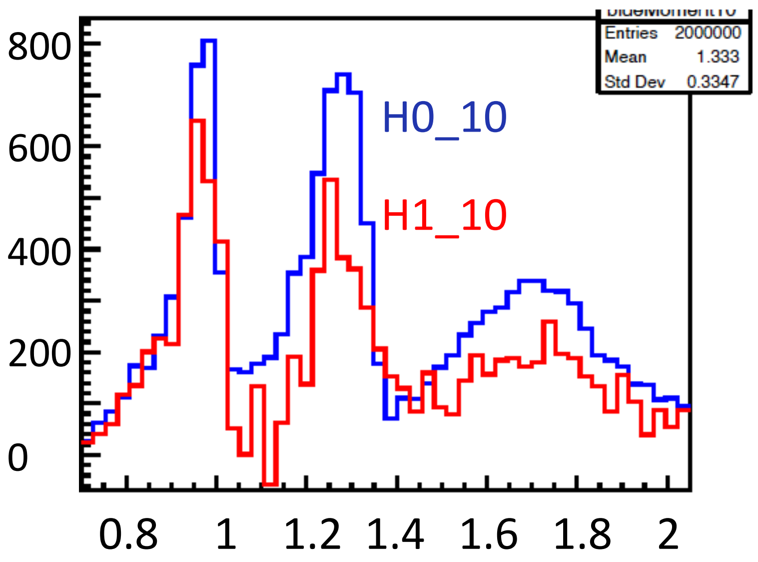






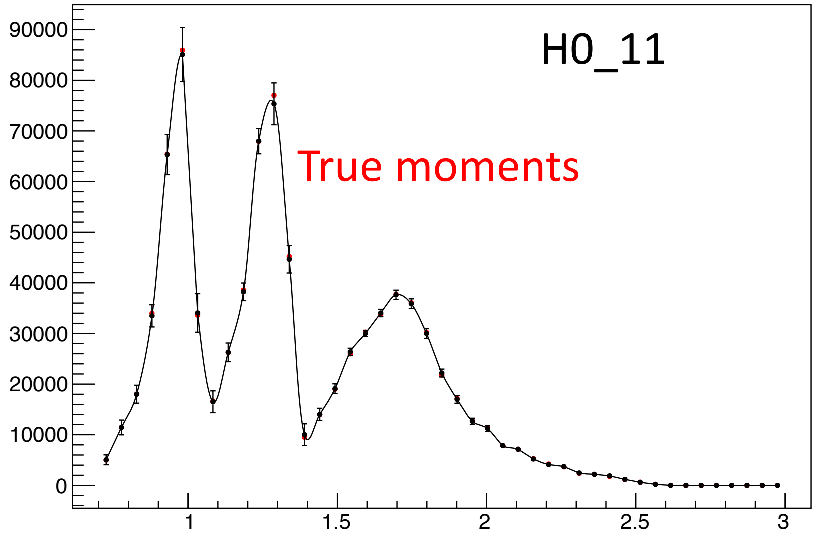


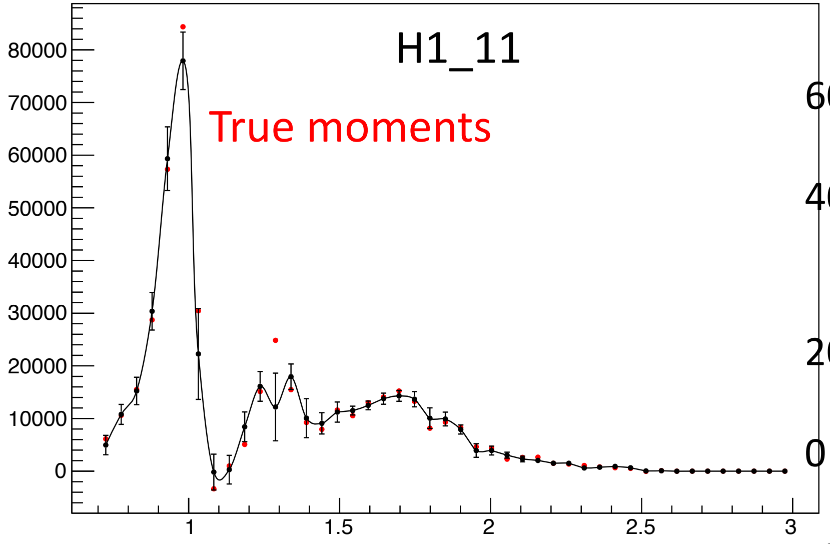


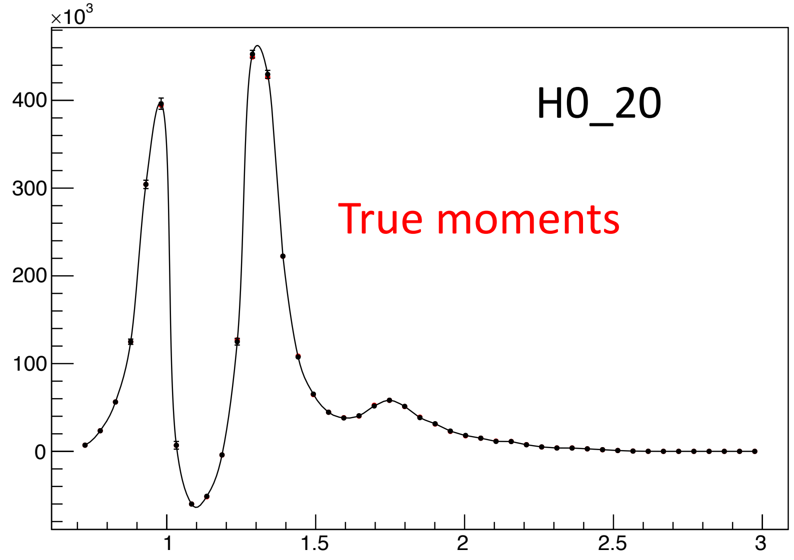


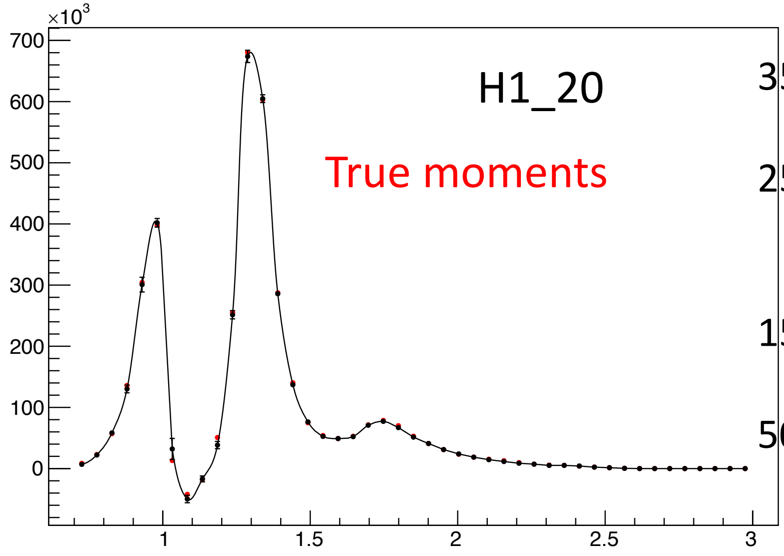
Polarized moments calculated with partial waves with uncertainties from bootstrapping

$0<t<0.3$ (GeV/c)^2 Calculated from fitted amplitudes , with bootstrapping uncert.









Unnormalized moments from Monte Carlo integration