$K^+K^-\pi^0$ update

Approximation

$f_1(x)f_2(x)$

Approximation

$$f_1(x)f_2(x) = \int f_1(y)f_2(y)\delta(y-x)dy$$

Approximation

$$f_1(x)f_2(x) = \int f_1(y)f_2(y)\delta(y-x)dy = Lim_{a\to 0} \int f_1(y)f_2(y)g(y-x)dy$$

Approximation

$$f_1(x)f_2(x) = \int f_1(y)f_2(y)\delta(y-x)dy = Lim_{a\to 0}\int f_1(y)f_2(y)g(y-x)dy$$

since $Lim_{a\to 0}g(y - x) = \delta(y-x)$,

Approximation

$$f_1(x)f_2(x) = \int f_1(y)f_2(y)\delta(y-x)dy = Lim_{a\to 0}\int f_1(y)f_2(y)g(y-x)dy$$

since $Lim_{a\to 0}g(y - x) = \delta(y-x)$, where $g = \frac{1}{|a|\sqrt{\pi}}e^{-(x/a)^2}$

Approximation

$$f_1(x)f_2(x) = \int f_1(y)f_2(y)\delta(y-x)dy = Lim_{a\rightarrow 0}\int f_1(y)f_2(y)g(y-x)dy$$

since $Lim_{a\rightarrow 0} g(y - x) = \delta(y-x)$, where $g = \frac{1}{|a|\sqrt{\pi}}e^{-(x/a)^2}$

so $f_1(x)f_2(x) \cong \int f_1(y)f_2(y)g(y-x)dy$

Approximation

$$f_1(x)f_2(x) = \int f_1(y)f_2(y)\delta(y-x)dy = Lim_{a\to 0}\int f_1(y)f_2(y)g(y-x)dy$$

since $Lim_{a\to 0} g(y - x) = \delta(y-x)$, where $g = \frac{1}{|a|\sqrt{\pi}}e^{-(x/a)^2}$

so $$f_1(x)f_2(x) \cong \int f_1(y)f_2(y)g(y-x)dy \quad (1)$$

Approximation I want, but I do not know how to implement in AmpTools

ASU

Approximation

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Approximation I want, but I do not know how to implement in AmpTools

Another form

$$f_1(x)f_2(x) = \int f_1(y_1)\delta(y_1-x)dy_1 \int f_2(y_2)\delta(y_2-x)dy_2$$

ASU

Approximation

$$f_1(x)f_2(x) = \int f_1(y)f_2(y)\delta(y-x)dy = Lim_{a\to 0}\int f_1(y)f_2(y)g(y-x)dy$$

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Approximation I want, but I do not know how to implement in AmpTools

Another form

$$f_1(x)f_2(x) = \int f_1(y_1)\delta(y_1-x)dy_1 \int f_2(y_2)\delta(y_2-x)dy_2$$

Now put in same type of approximation for the Dirac delta functions:

$$f_1(x)f_2(x) \cong \int f_1(y_1)g(y_1-x)dy_1 \int f_2(y_2)g(y_2-x)dy_2$$

ASU

Approximation

$$f_1(x)f_2(x) = \int f_1(y)f_2(y)\delta(y-x)dy = Lim_{a\to 0}\int f_1(y)f_2(y)g(y-x)dy$$

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so $$f_1(x)f_2(x) \cong \int f_1(y)f_2(y)g(y-x)dy \qquad (1)$$

Approximation I want, but I do not know how to implement in AmpTools

---

Another approximation

$$f_1(x)f_2(x) = \int f_1(y_1)\delta(y_1-x)dy_1 \int f_2(y_2)\delta(y_2-x)dy_2$$

Now put in same type of approximation for the Dirac delta functions:

$$f_1(x)f_2(x) \cong \int f_1(y_1)g(y_1-x)dy_1 \int f_2(y_2)g(y_2-x)dy_2 \qquad (2)$$

I know how to implement this approximation in AmpTools

Smear parameter for different type of approximations

Want: $f_1(x)f_2(x) \cong \int f_1(y)f_2(y)g(y-x)dy$

Smear parameter for different type of approximations

Want: $f_1(x)f_2(x) \cong \int f_1(y)f_2(y)g(y-x)dy$

Have: $f_1(x)f_2(x) \cong \iint f_1(y_1)f_2(y_2)g(y_1-x)g(y_2-x)dy_1dy_2$

Smear parameter for different type of approximations

Same smear parameters?

Want: $f_1(x)f_2(x) \cong \int f_1(y)f_2(y)g(y-x)dy$

Have: $f_1(x)f_2(x) \cong \iint f_1(y_1)f_2(y_2)g(y_1-x)g(y_2-x)dy_1dy_2$

Smear parameter for different type of approximations

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Have: $f_1(x)f_2(x) \cong \iint f_1(y_1)f_2(y_2)g(y_1-x)g(y_2-x)dy_1dy_2$

If a Dirac delta, $\delta(y_1\text{-}y_2)$, introduced prior to approximation in Eqn. 2 :

$$f_1(x)f_2(x) = \iint f_1(y_1)f_2(y_2)\delta(y_1-x)\delta(y_2-x)\delta(y_1-y_2)dy_1dy_2$$

ASU

Smear parameter for different type of approximations

Same smear parameters?

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$$= \iint f_1(y)f_2(y)\delta(y-x)dy$$

Smear parameter for different type of approximations

Same smear parameters?

Want: $f_1(x)f_2(x) \cong \int f_1(y)f_2(y)g(y-x)dy$

Have: $f_1(x)f_2(x) \cong \iint f_1(y_1)f_2(y_2)g(y_1-x)g(y_2-x)dy_1dy_2$

If a Dirac delta, $\delta(y_1\text{-}y_2)$, introduced prior to approximation in Eqn. 2 :

$$
\begin{aligned}
f_1(x)f_2(x) &= \iint f_1(y_1)f_2(y_2)\delta(y_1-x)\delta(y_2-x)\delta(y_1-y_2)dy_1dy_2 \\
&= \iint f_1(y)f_2(y)\delta(y-x)dy \\
&\cong \int f_1(y)f_2(y)g(y-x)dy
\end{aligned}
$$

Smear parameter for different type of approximations

Want: $f_1(x)f_2(x) \cong \int f_1(y)f_2(y)\boxed{g(y-x)}dy$

Have: $f_1(x)f_2(x) \cong \iint f_1(y_1)f_2(y_2)\boxed{g(y_1-x)}\boxed{g(y_2-x)}dy_1dy_2$

Same smear parameters?

If a Dirac delta, $\delta(y_1\text{-}y_2)$, introduced prior to approximation in Eqn. 2 :

$$f_1(x)f_2(x) = \iint f_1(y_1)f_2(y_2)\delta(y_1-x)\delta(y_2-x)\delta(y_1-\delta y_2)dy_1dy_2$$

$$= \iint f_1(y)f_2(y)\delta(y-x)dy$$

$$\cong \int f_1(y)f_2(y)g(y-x)dy$$

If a Dirac delta, $\delta(y_1\text{-}y_2)$, introduced in Eqn. 2 after approximation :

$$f_1(x)f_2(x) \cong \int f_1(y)f_2(y)g(y-x)\boxed{g(y-x)}dy$$

Extra factor of $g(y-x)$

ASU

$g^{1/2}$

Note: $g^{1/2} \propto e^{-\frac{x^2}{4\sigma^2}}$ and other than an overall constant is the same as using g and taking $\sigma_{1/2} = \sigma\sqrt{2}$

$g^{1/2}$

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- For now, I am using $g^{1/2}$ the function that is convoluted over Breit-Wigner amplitudes.

ASU

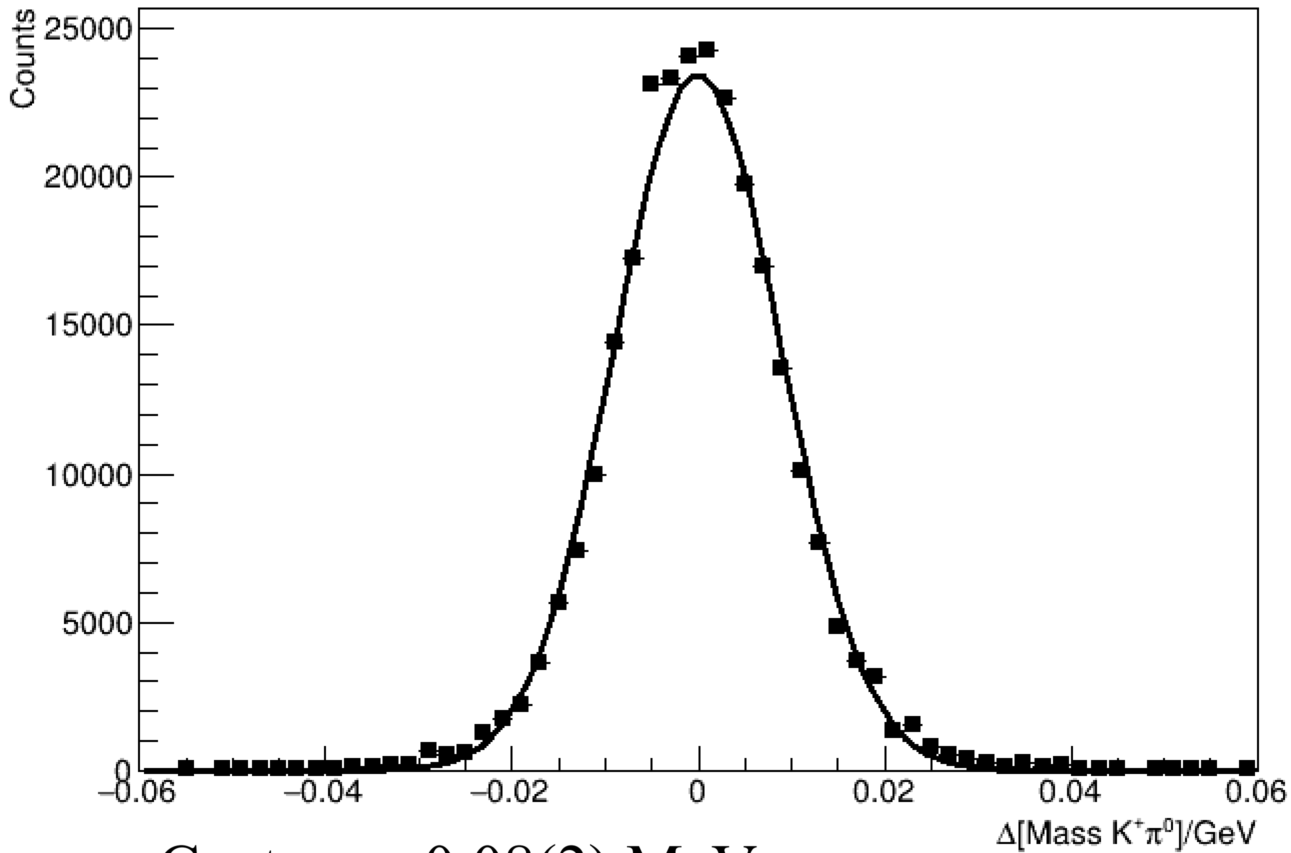
$g^{1/2}$

Note: $g^{1/2} \propto e^{-\frac{x^2}{4\sigma^2}}$ and other than an overall constant is the same as using g and taking $\sigma_{1/2} = \sigma\sqrt{2}$

- For now, I am using $g^{1/2}$ the function that is convoluted over Breit-Wigner amplitudes.

- If I continue down this road, I anticipate changing from $g^{1/2}$ to g

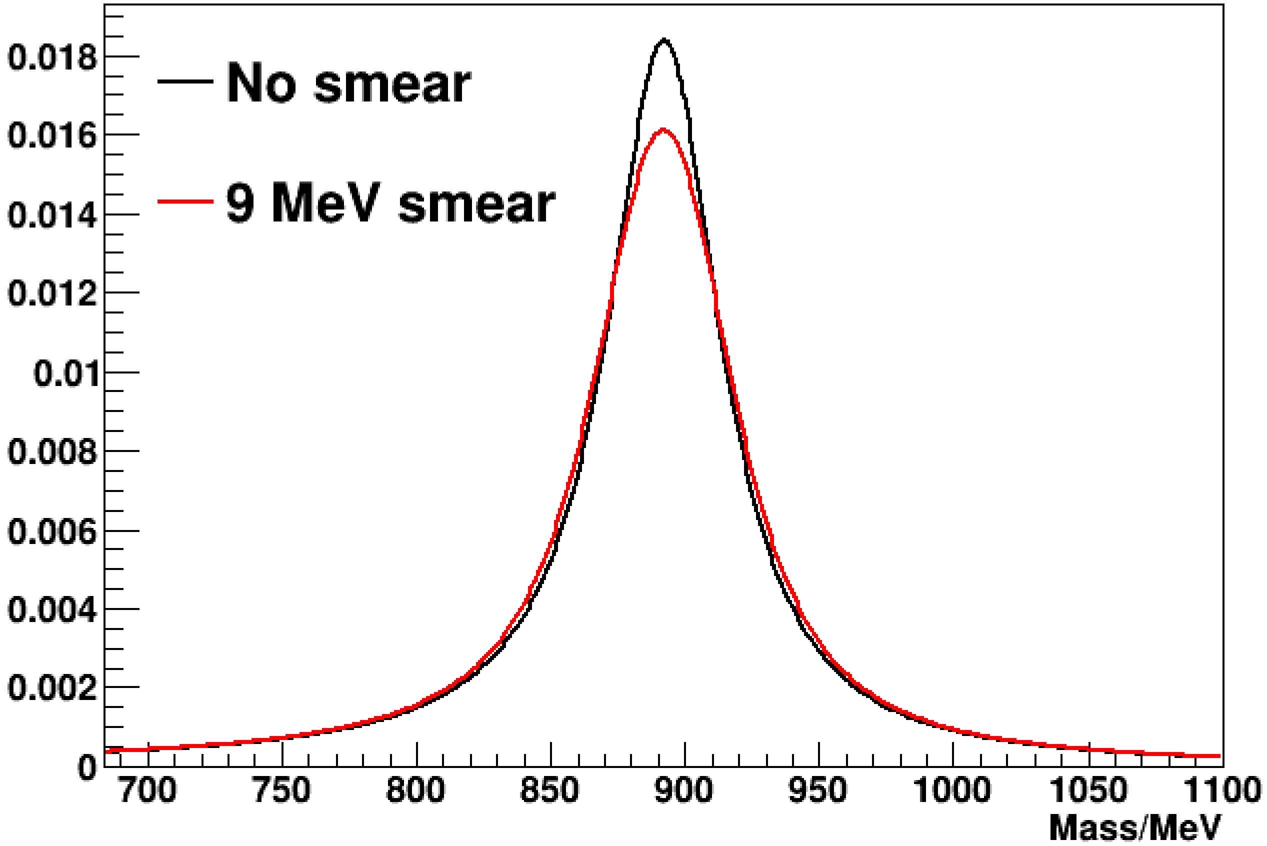
$\Delta[\text{Mass}(K^{+}\pi^{0})]$ fit to gaussian



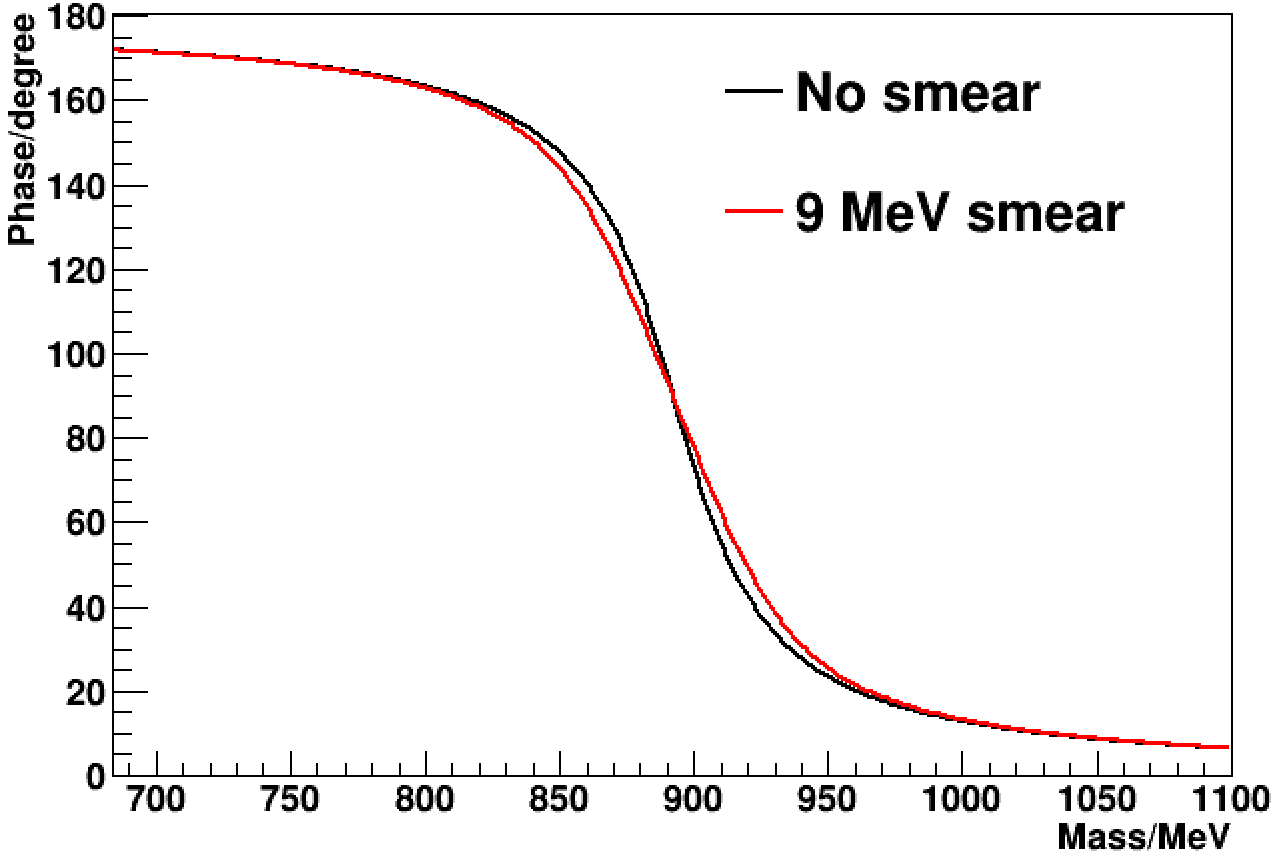
Center = -0.08(2) MeV

σ = 9.01(2) MeV

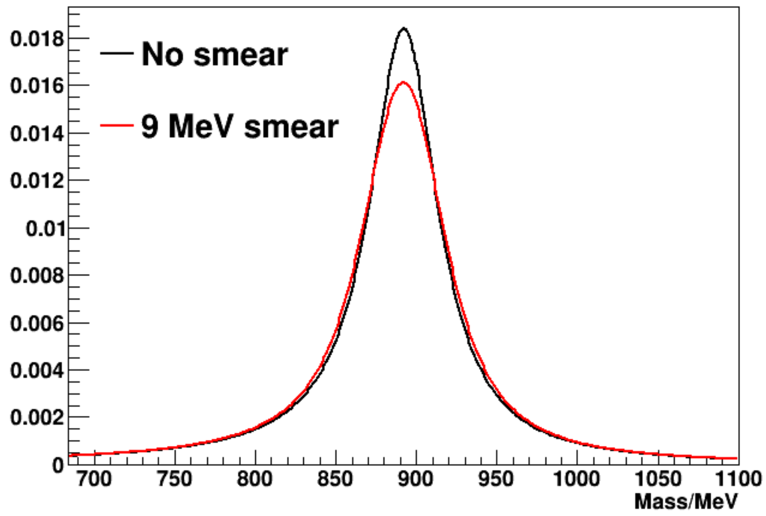
Mass distribution of isobar

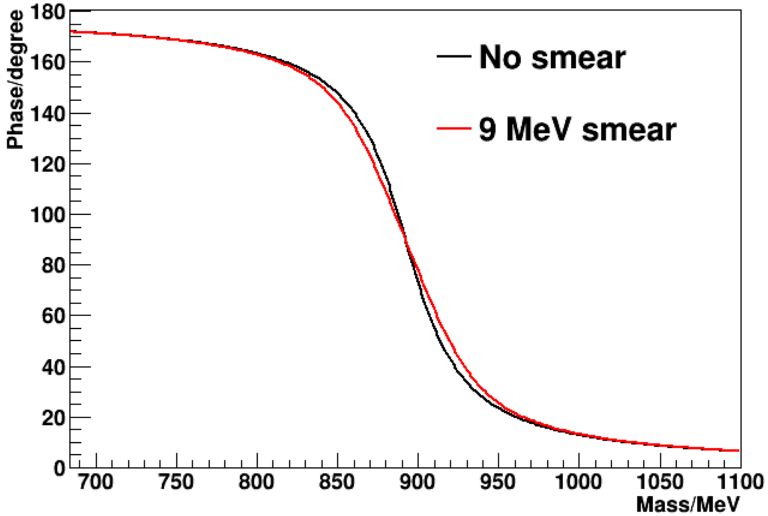


Phase distribution of isobar



Breit-Wigner with/without smearing

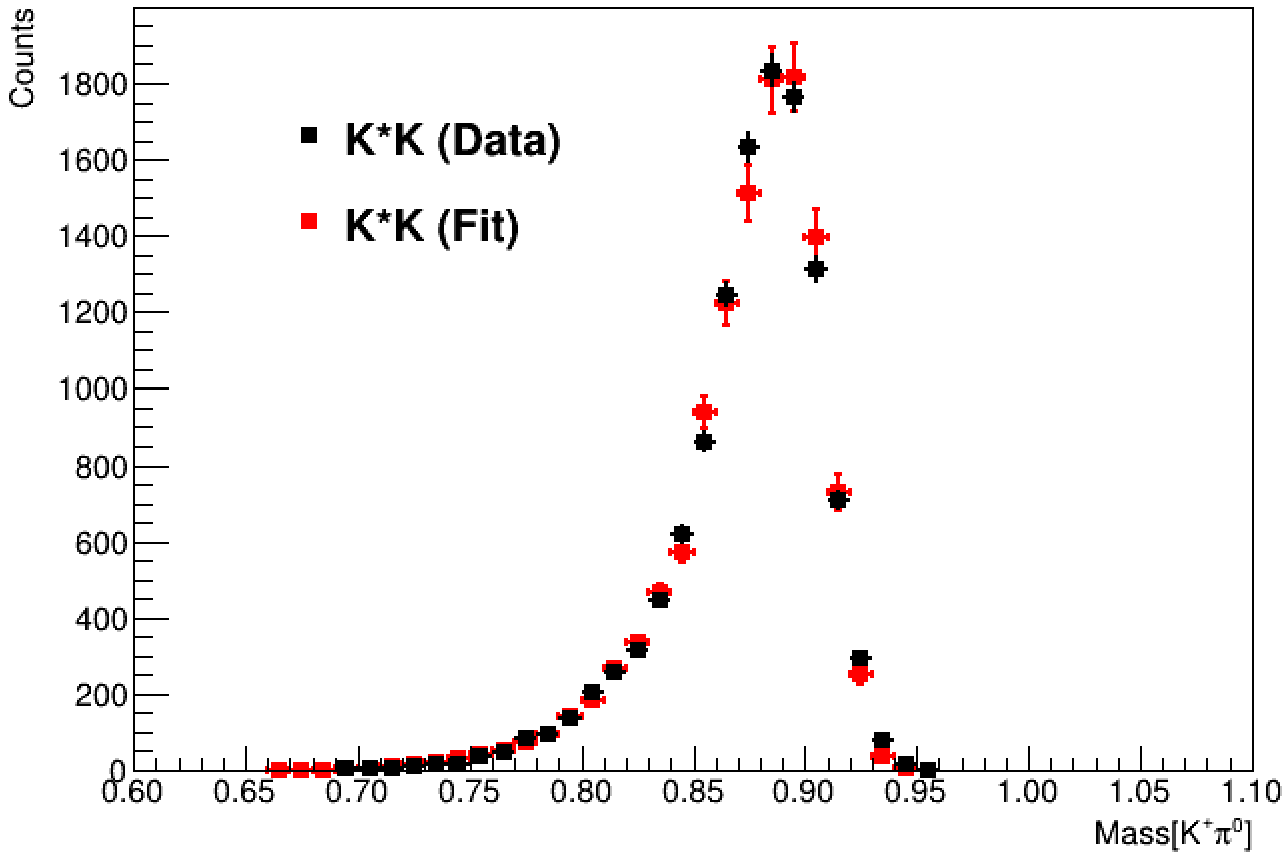




Data

- All data shown in this presentation are from simulation

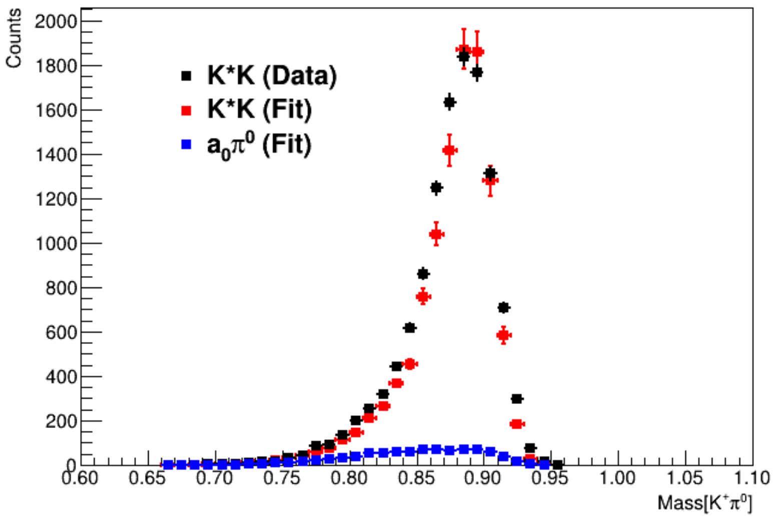
Fit without a_0 term to establish smear parameter



$\sigma_{1/2} = 8.2(4)$ MeV $\Rightarrow$ $\sigma = 11.6(6)$ MeV

K^* isobar fits (no angular information fit)

Without gaussian convolution



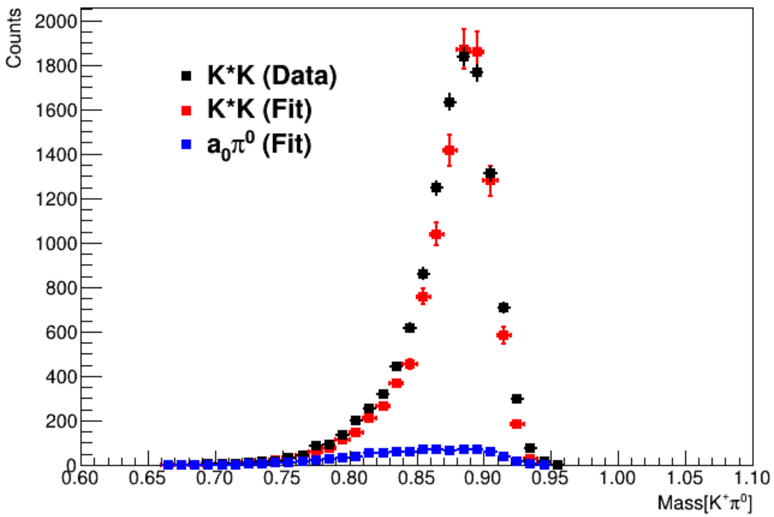
Fraction $K^*K = 0.92(3)$

Fraction $a_0\pi^0 = 0.07(1)$

ASU

K^* isobar fits (no angular information fit)

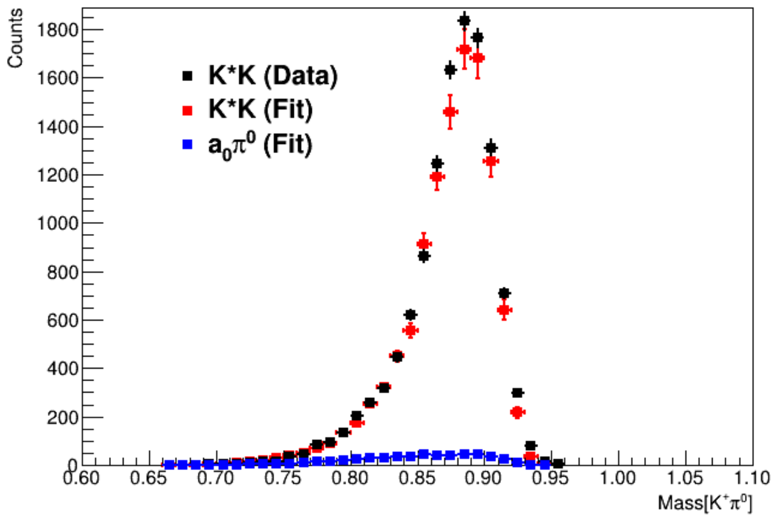
Without gaussian convolution



Fraction $K^*K = 0.92(3)$

Fraction $a_0\pi^0 = 0.07(1)$

With gaussian convolution



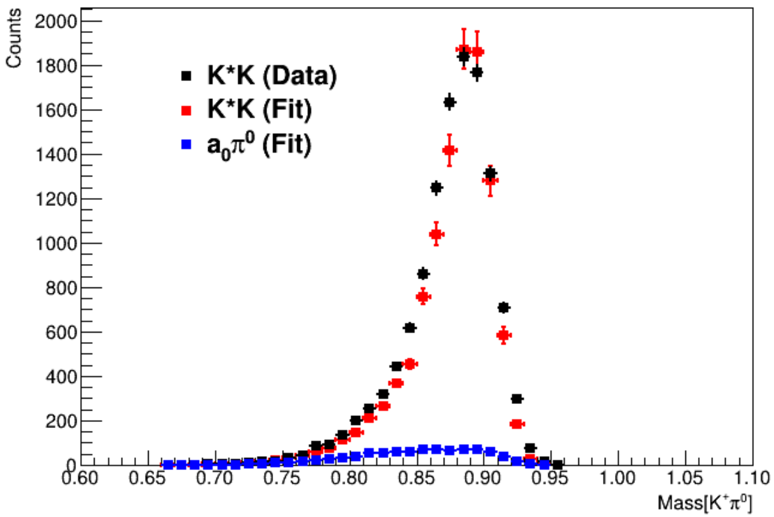
Fraction $K^*K = 0.98(3)$

Fraction $a_0\pi^0 = 0.04(1)$

ASU

K^* isobar fits (no angular information fit)

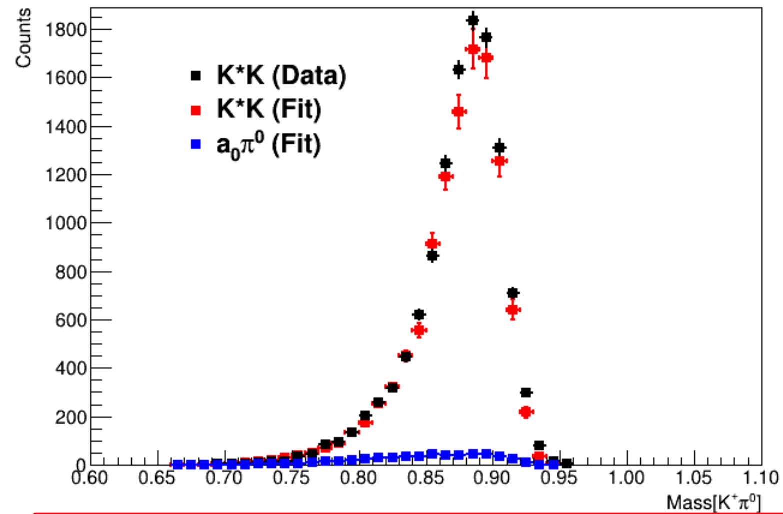
Without gaussian convolution



Fraction $K^*K = 0.92(3)$

Fraction $a_0\pi^0 = 0.07(1)$

With gaussian convolution

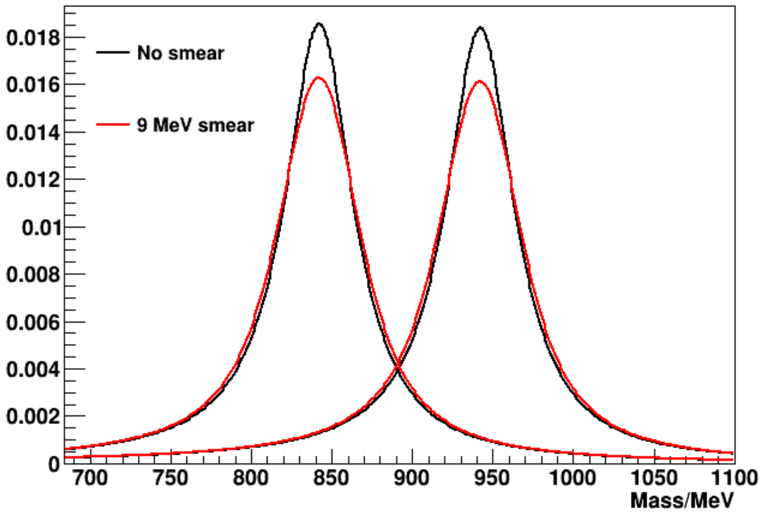


Fraction $K^*K = 0.98(3)$

Fraction $a_0\pi^0 = 0.04(1)$

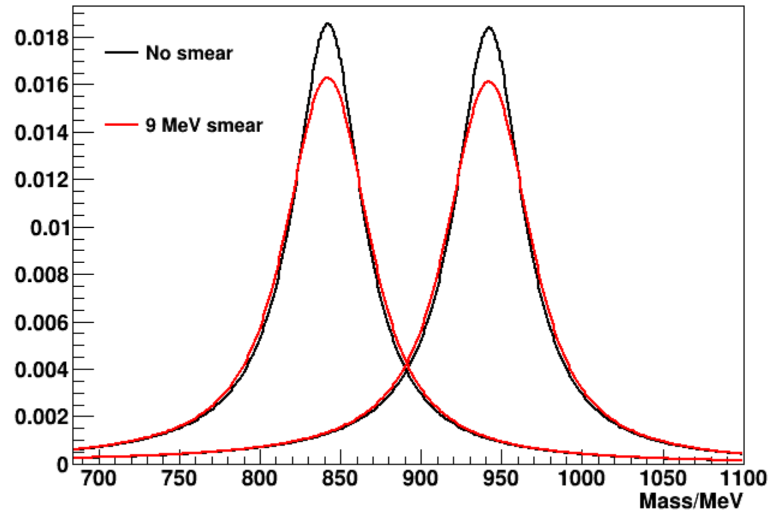
Convolution made fit better ☺

Double Breit-Wigners

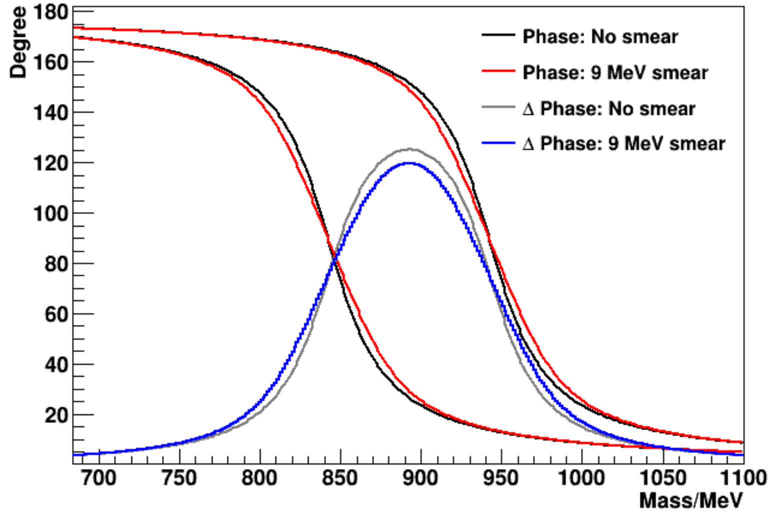


- 100 MeV apart

Double Breit-Wigners

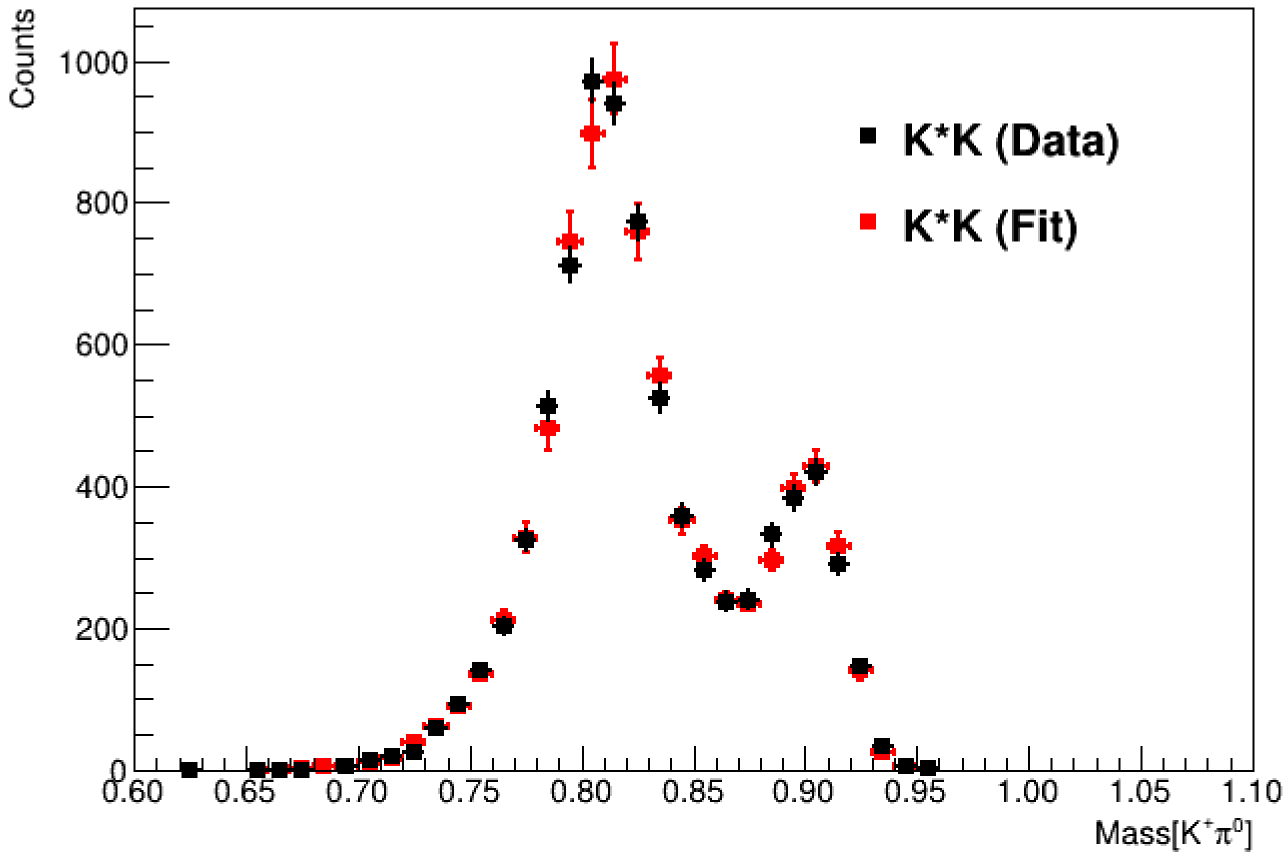


- 100 MeV apart



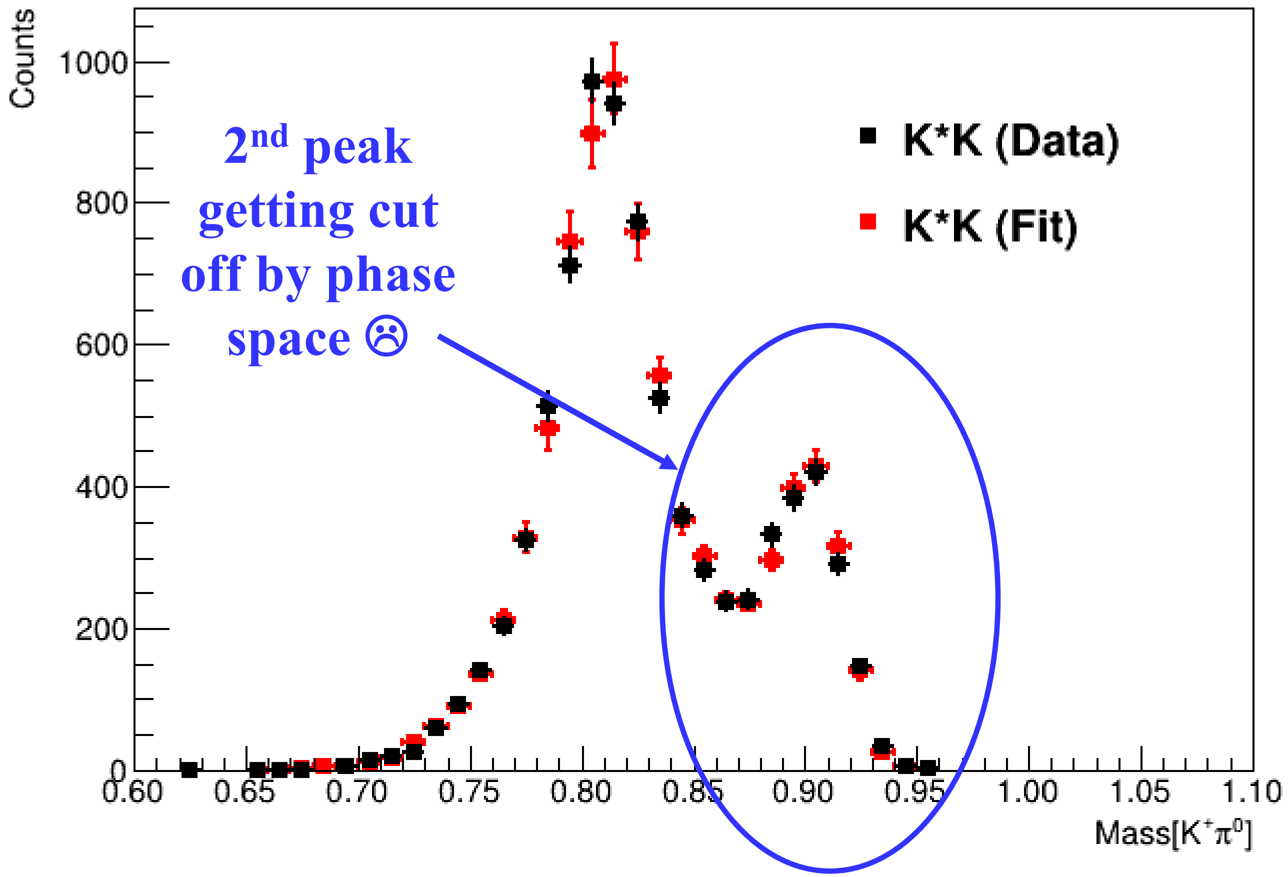
- Δ Phase very similar ☺

Fit without a_0 term to establish smear parameter



$\sigma_{1/2} = 6.2(5)$ MeV $\Rightarrow$ $\sigma = 8.8(8)$ MeV

Fit without a_0 term to establish smear parameter

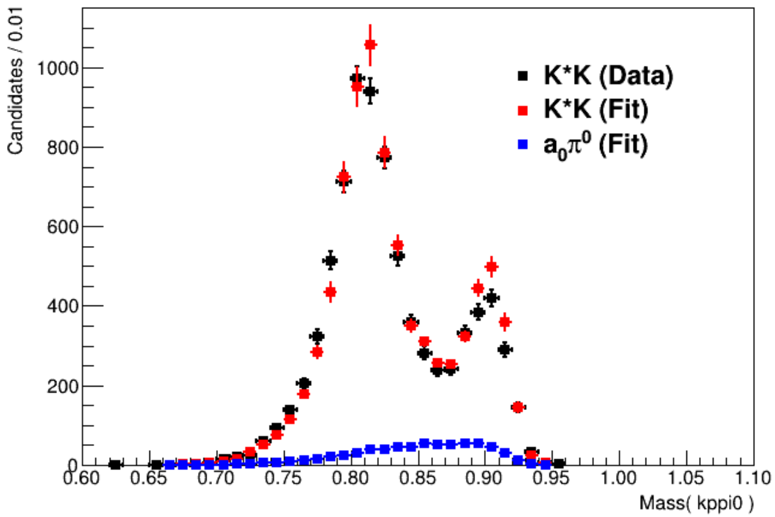


$$\sigma_{1/2} = 6.2(5)\ \text{MeV} \Rightarrow \sigma = 8.8(8)\ \text{MeV}$$

ASU

K^* isobar fits (no angular information fit)

Without convolution

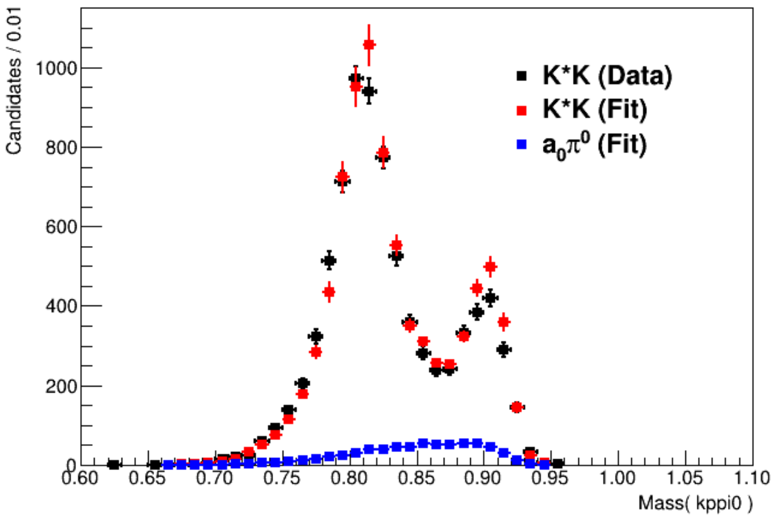


Fraction $K^*K = 0.98(4)$

Fraction $a_0\pi^0 = 0.05(1)$

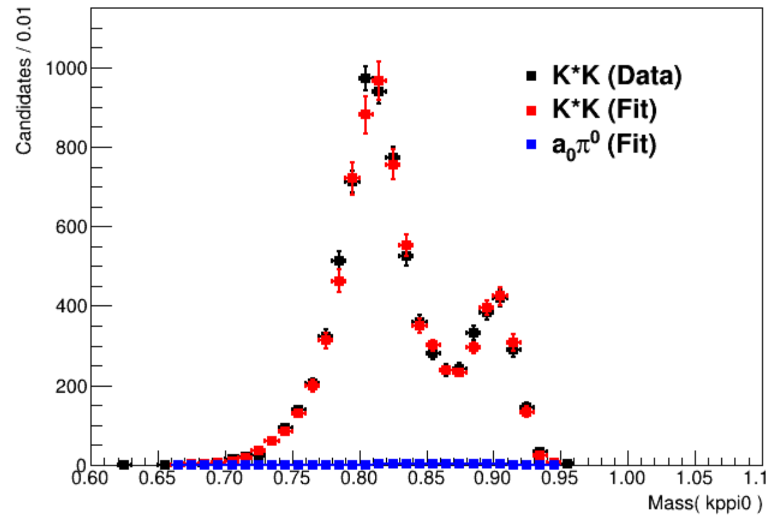
K^* isobar fits (no angular information fit)

Without convolution



Fraction $K^*K = 0.98(4)$
Fraction $a_0\pi^0 = 0.05(1)$

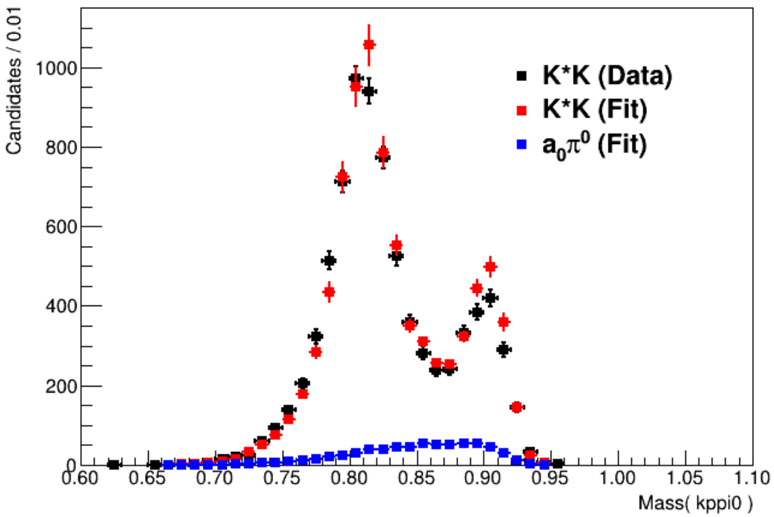
With convolution



Fraction $K^*K = 0.97(4)$
Fraction $a_0\pi^0 = 0.002(1)$

K^* isobar fits (no angular information fit)

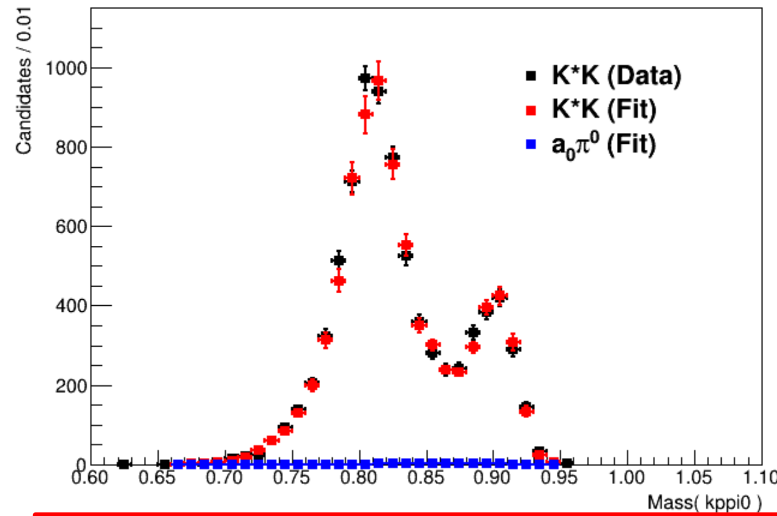
Without convolution



Fraction $K^*K = 0.98(4)$

Fraction $a_0\pi^0 = 0.05(1)$

With convolution



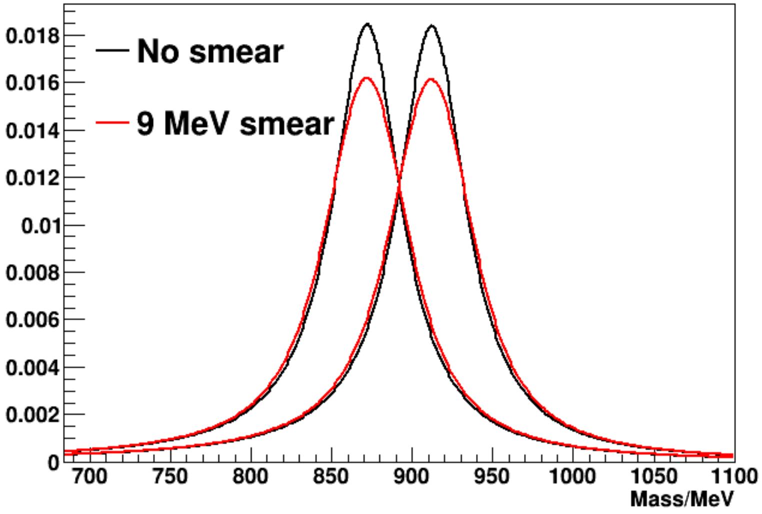
Fraction $K^*K = 0.97(4)$

Fraction $a_0\pi^0 = 0.002(1)$

Convolution made fit better ☺

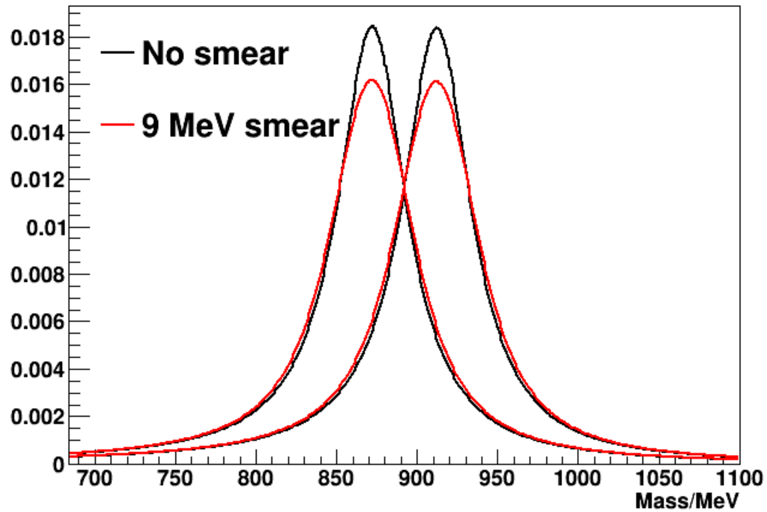
ASU

Double Breit-Wigners

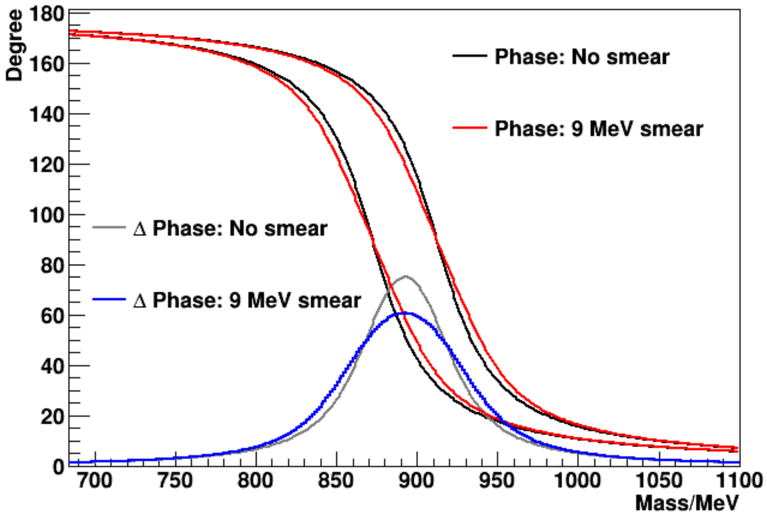


- 40 MeV apart

Double Breit-Wigners

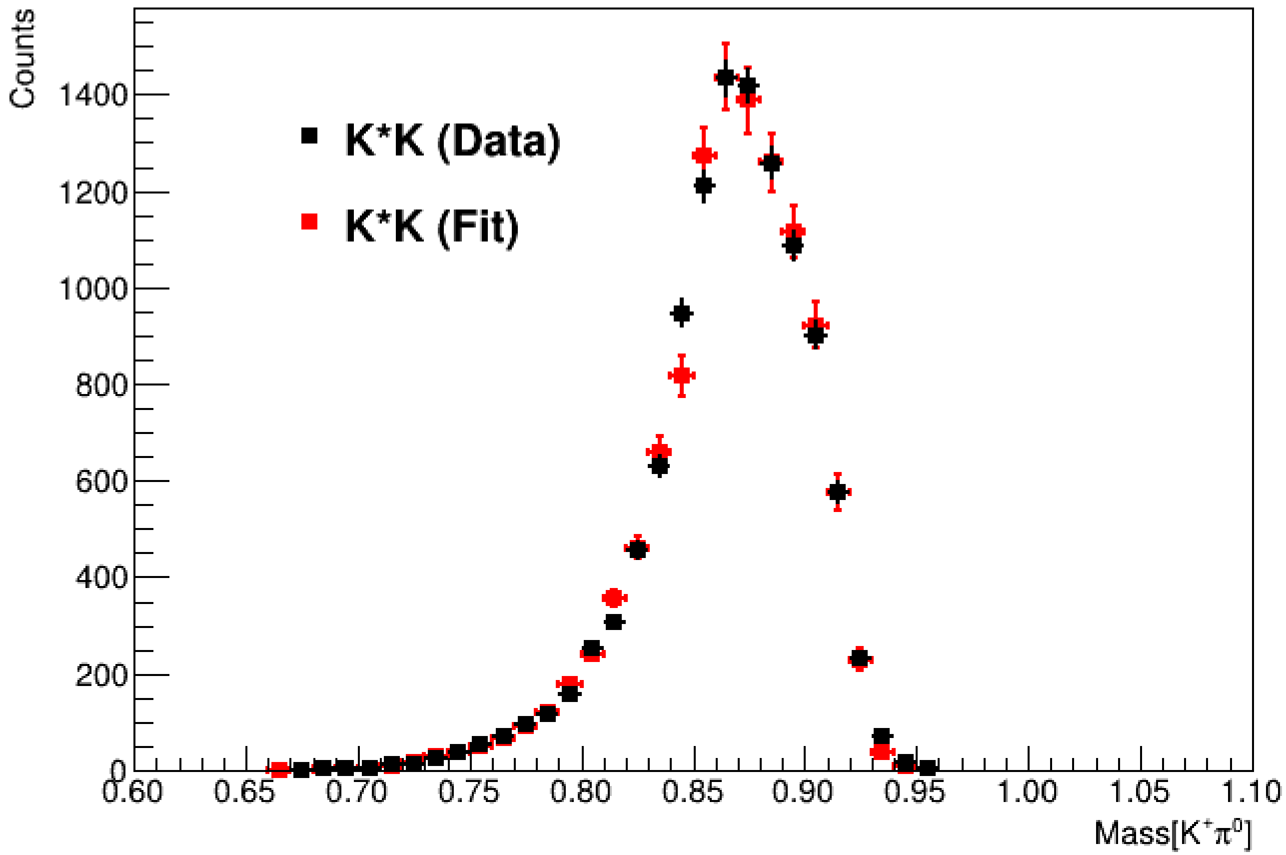


- 40 MeV apart



- Δ Phase not as similar as when peaks are set 100 MeV apart ☹

Fit without a_0 term to establish smear parameter

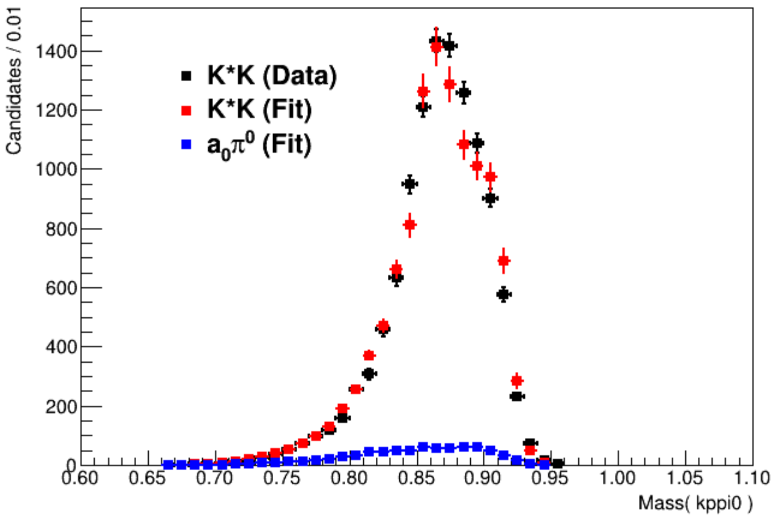


$\sigma_{1/2} = 5.2(8)$ MeV $\Rightarrow$ $\sigma = 7(1)$ MeV

ASU

K^* isobar fits (no angular information fit)

Without convolution

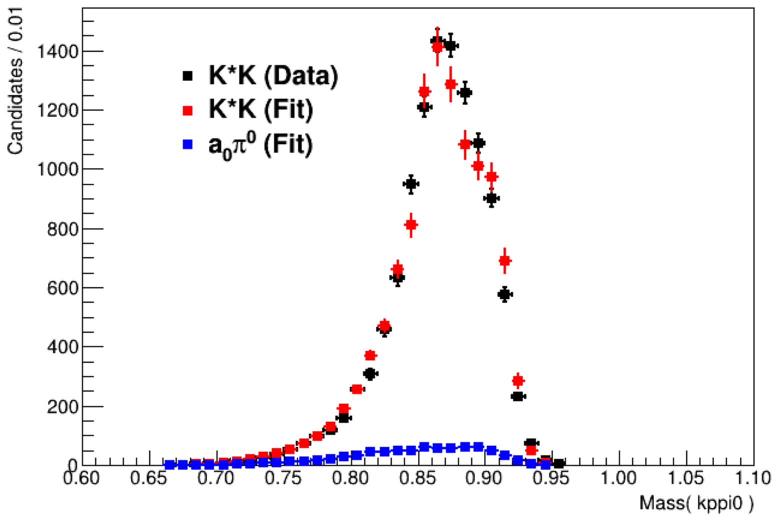


Fraction $K^*K = 1.00(3)$

Fraction $a_0\pi^0 = 0.05(2)$

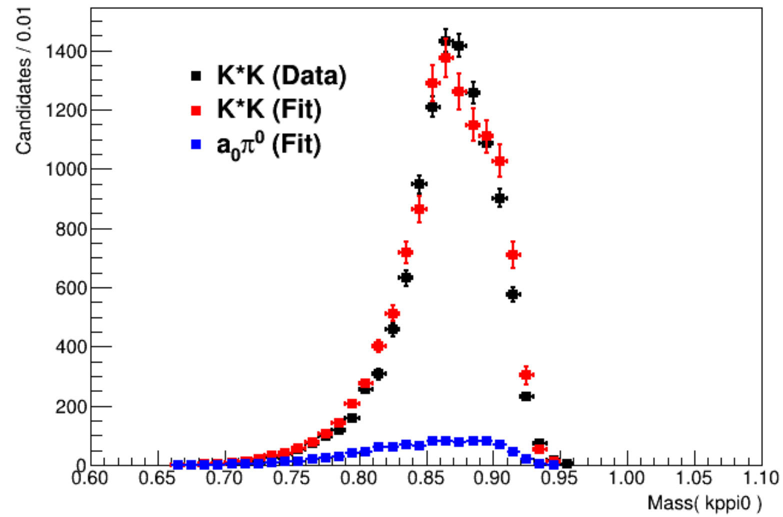
K^* isobar fits (no angular information fit)

Without convolution



Fraction K^*K = 1.00(3)
Fraction $a_0\pi^0$ = 0.05(2)

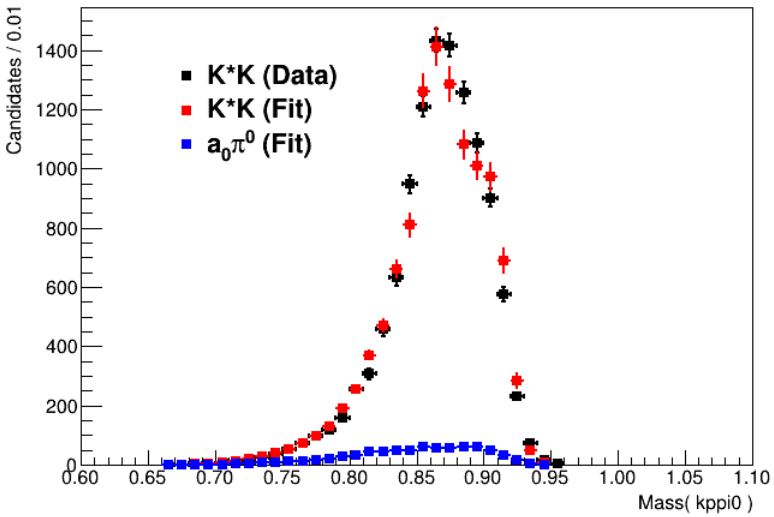
With convolution



Fraction K^*K = 1.05(4)
Fraction $a_0\pi^0$ = 0.07(3)

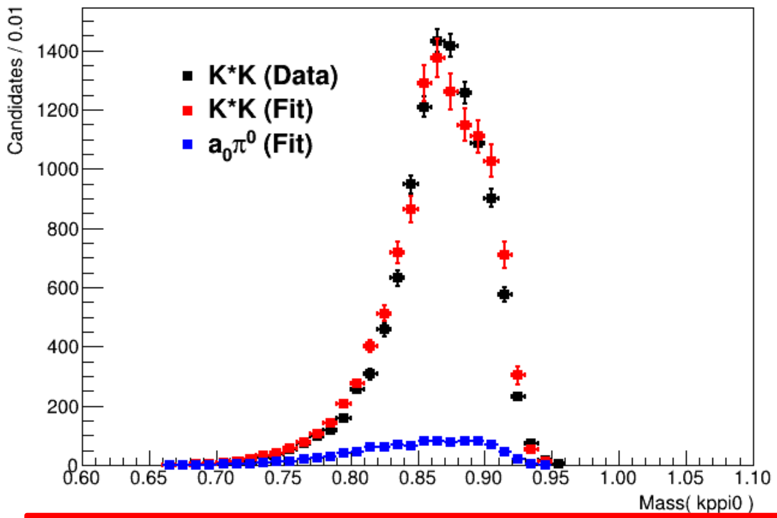
K^* isobar fits (no angular information fit)

Without convolution



Fraction $K^*K = 1.00(3)$
Fraction $a_0\pi^0 = 0.05(2)$

With convolution



Fraction $K^*K = 1.05(4)$
Fraction $a_0\pi^0 = 0.07(3)$

This time, convolution made fit worse ☹

Summary

- Convolution can make fit better but not always
- When Breit-Wigner isobars are close together, the convolution causes fit results to be less reliable
- When Breit-Wigner isobars are far apart (100MeV apart with Γ = 52 MeV), the convolution is helpful in obtaining a good fit.

Potential next steps?

- Have only looked at K^{+*}, but actual data will have $K^{+*}K^{-}$ and $K^{+}K^{-*}$ in combined isospin states with I=0 or I=1.
  - Will need to generate I=0, and I=1 states.
- For the actual $K^{*}(892)$ isobar ($K\pi$ type), the isobar interferences will be primarily with $K^{+}K^{-}$ type isobars (e.g. a_0) and will have to be studied.

ASU

Title