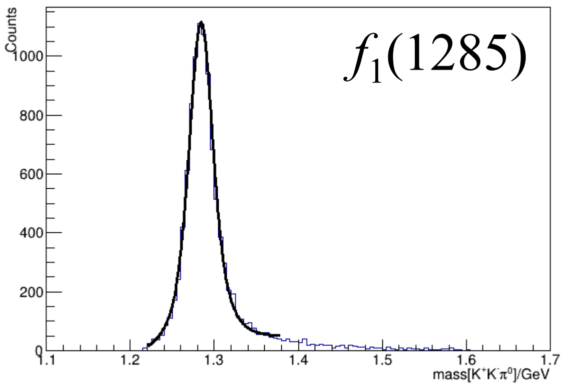
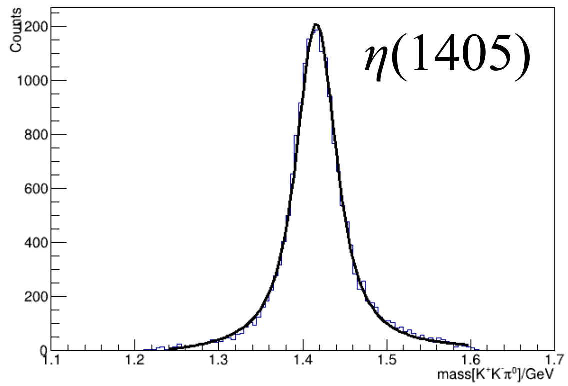
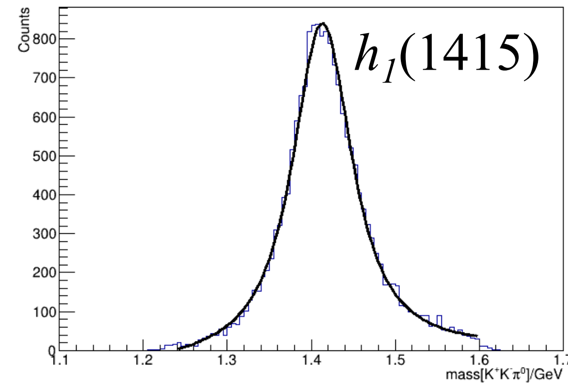
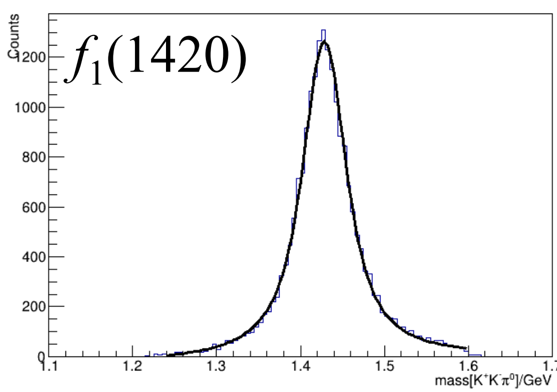
# $K^+K^-\pi^0$ update

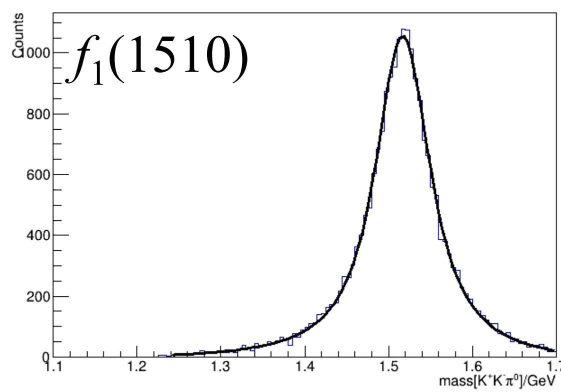
# Monte Carlo peak fits



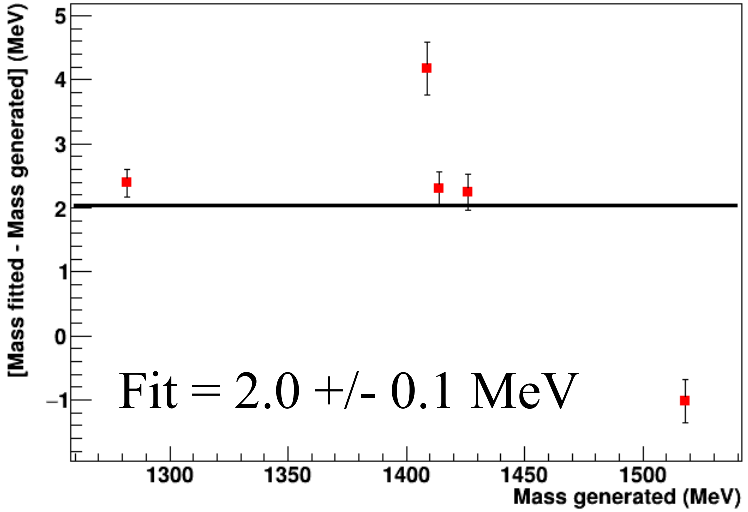




- Each mass spectrum was fit to voigtian line shape

# Results of Monte Carlo peak fits

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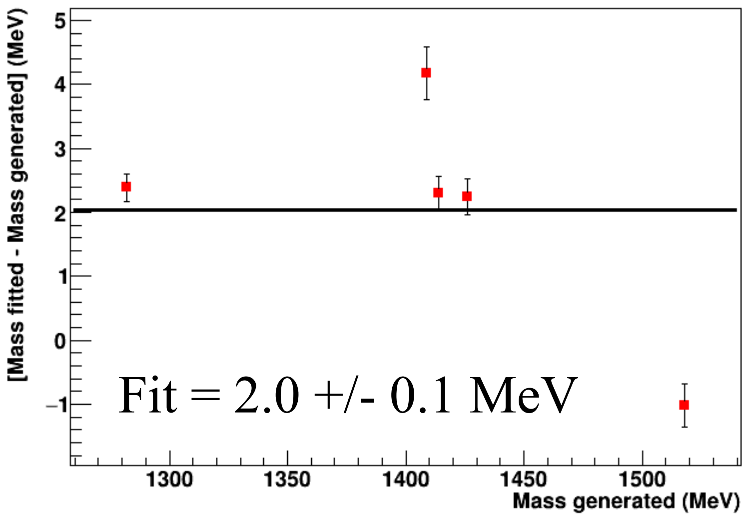


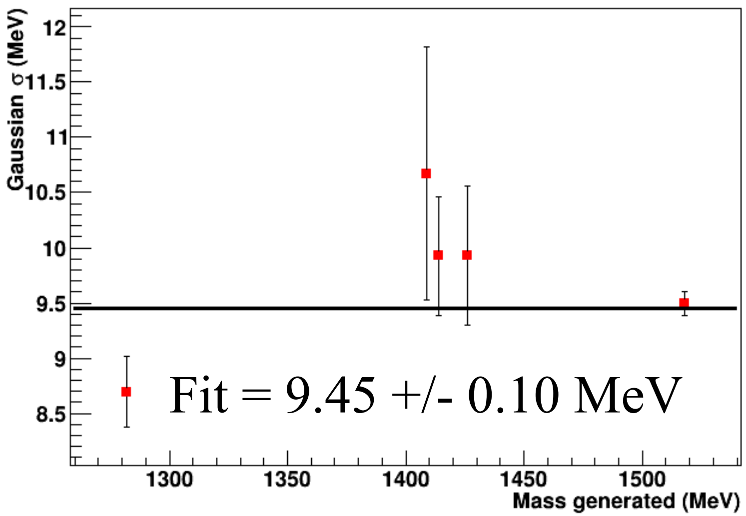


- Reconstructed masses are systematically high by about 2 MeV
- Gaussian broadening ($\sigma$) of Voigtian line shape is about 9.45 MeV

# Intensity

- Looking at the intensity in two different ways:
  - Salgado-Weygand
  - JPAC

# Intensity

The intensity can be written as [1] :

$$I = \sum_{\text{ext spins}} \left| \left\langle out \middle| \hat{T} \middle| in \right\rangle \right|^2$$

[1] Carlos Salgado and Dennis Weygand, arXiv:1310.7498v2

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Define the spin density matrix $\rho$ such that:

$$|in\rangle\langle in| = \sum_{i,j} \rho_{i,j} = \sum_{i,j} |i\rangle\langle j|$$

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Insert full set of partial waves

$$1 = \sum_X |X\rangle\langle X|$$

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where $\langle out | \hat{T}_d | X\rangle$ is the partial wave amplitude

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# Partial wave amplitude

$$X^{j}_{lms} =$$

$$\frac{m_0\Gamma}{{m_0}^2 - m^2 - i m_0\Gamma} a_{jlsm} \sum_\lambda D^{J*}_{m\lambda}(\varphi_{GJ}, \theta_{GJ}) D^{s*}_{\lambda 0}(\varphi_h, \theta_h) \langle l0s\lambda | J\lambda \rangle,$$

where $a_{jlsm}$ are the coefficients of the fit, and the Breit-Wigner factor is for the potential isobar

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ASU

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- Operators:
  - Parity operator = $\Pi$
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- Reflectivity quantum number $\epsilon$, with
  - $\epsilon = \pm 1$ for bosons
  - $\epsilon = \pm i$ for fermions

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Let $m$ be angular momentum $z$-projection, $\epsilon$ be reflectivity and $a$ be all other quantum numbers.

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States of definite reflectivity:

- $|\, \epsilon, a, m> = \Theta(m)[|a, m> + \epsilon P(-1)^{J-m}|a, -m>]$,

where

  - $\Theta(m) = \frac{1}{\sqrt{2}}$, if $m > 0$
  - $\Theta(m) = \frac{1}{2}, if\ m = 0$
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- When $m = 0$ we have $|\,\epsilon, a, 0> =$½ $[1 + \epsilon P(-1)^{J}]|a, 0>$
  - Only non-zero when $\epsilon = P(-1)^{J}$, or
    - $\epsilon = +P$, for $J =$ even
    - $\epsilon = -P$, for $J =$ odd

# Natural parity and Naturality

Definitions:

- Natural parity if $P = (-1)^J$ and unnatural parity if $P = -(-1)^J$ .
  So:
  - Scalars and vectors are Natural
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  - Scalars and vectors are Natural
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- Naturality ($N$) of the exchanged particle is $N = P(-1)^J$ , with Natural when $N = +1$ and Unnatural when $N = -1$.

# Intensity (JPAC method 2-pseudoscalars)

The intensity in the helicity basis can be written as [2]

$$I(\Omega, \mathscr{P}, \Phi) = \sum_{\lambda_1,\lambda_2} \sum_{\lambda,\lambda'} T_{\lambda;\lambda_1,\lambda_2}(\Omega) \rho^{\gamma}_{\lambda,\lambda'}(\mathscr{P}, \Phi) T^{*}_{\lambda';\lambda_1,\lambda_2}(\Omega). \quad (76)$$

[2] Carlos Salgado and Vincent Mathieu, https://halldweb.jlab.org/DocDB/0045/004599/001/photonPWA_FINAL.pdf
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$$T^{\ell}_{\lambda m;\lambda_1\lambda_2} \simeq -P(-1)^J(-1)^m T^{\ell,s}_{-\lambda-m;\lambda_1\lambda_2}, \qquad \text{(C8)}$$

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ASU

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Trading photon-helicity index for photon-reflectivity and define the reflectivity basis

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Note: Only for 2-pseudoscalar production

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# Intensity (Justin Stevens)

Justin took the JPAC method written for 2-pseudoscalar production and expanded it for vector pseudoscalar

Modified → $$^{(\epsilon)}T^l_m = \frac{1}{2}\left(T^l_{+1,m} - \epsilon(-1)^m T^l_{-1,-m}\right) \quad (5)$$

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To include the reflectivity of the meson resonance

$$^{(\epsilon)}T^{i}_{m} = \frac{1}{2}\left(T^{i}_{+1,m} - \epsilon\tau_{i}(-1)^{m}T^{i}_{-1,-m}\right) \quad (6)$$

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And once the dust settles →

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# Intensity (Justin Stevens)

$$I(\Phi,\Omega,\Omega_H) = 2\kappa \sum_k$$

$$\Bigg\{(1-P_\gamma)\left[\left|\sum_{i_N,m}[J_i^N]^{(+)}_{m,k} Im(Z) + \sum_{i_U,m}[J_i^U]^{(-)}_{m,k} Im(Z)\right|^2 + \left|\sum_{i_N,m}[J_i^N]^{(-)}_{m,k} Re(Z) + \sum_{i_U,m}[J_i^U]^{(+)}_{m,k} Re(Z)\right|^2\right] +$$

$$(1+P_\gamma)\left[\left|\sum_{i_N,m}[J_i^N]^{(-)}_{m,k} Im(Z) + \sum_{i_U,m}[J_i^U]^{(+)}_{m,k} Im(Z)\right|^2 + \left|\sum_{i_N,m}[J_i^N]^{(+)}_{m,k} Re(Z) + \sum_{i_U,m}[J_i^U]^{(-)}_{m,k} Re(Z)\right|^2\right]\Bigg\}$$

The $[J_i^{N,U}]^{(\epsilon)}_{m,k}$ are the free complex parameters in the fit for a given reflectivity amplitude.

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$$(1+P_\gamma)\left[\left|\sum_{i_N,m}[J_i^N]^{(-)}_{m,k}Im(Z)+\sum_{i_U,m}[J_i^U]^{(+)}_{m,k}Im(Z)\right|^2+\left|\sum_{i_N,m}[J_i^N]^{(+)}_{m,k}Re(Z)+\sum_{i_U,m}[J_i^U]^{(-)}_{m,k}Re(Z)\right|^2\right]\Bigg\}$$

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where $Z^i_m(\Omega,\Omega_H) = e^{-i\Phi}X^i_m(\Omega,\Omega_H)$ is the phase-rotated decay amplitude and $\Phi$ is the angle between the production plane and the photon polarization

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Justin Stevens, https://halldweb.jlab.org/doc-private/DocDB/ShowDocument?docid=4858

# $K^*K$ decays

- $K^+$ has $I = ½$ and $I_z = + ½$ : $|\, ½ \,, +½ \,\rangle$
- $K^-$ has $I = ½$ and $I_z =$ - ½ : $|\, ½ \,, -½ \,\rangle$

For $I$=0 resonance $\rightarrow K^*K$ :

- $|\, 0 \,, 0 \,\rangle = \frac{1}{\sqrt{2}}[|\, ½ \,, +½ \,\rangle |\, ½ \,, -½ \,\rangle - |\, ½ \,, -½ \,\rangle |\, ½ \,, +½ \,\rangle]$
- $|\, 0 \,, 0 \,\rangle = \frac{1}{\sqrt{2}}[K^{+*}K^{-} - K^{-*}K^{+}]$

For $I$=1 resonance $\rightarrow K^*K$ :

- $|\, 1 \,, 0 \,\rangle = \frac{1}{\sqrt{2}}[|\, ½ \,, +½ \,\rangle |\, ½ \,, -½ \,\rangle + |\, ½ \,, -½ \,\rangle |\, ½ \,, +½ \,\rangle]$
- $|\, 1 \,, 0 \,\rangle = \frac{1}{\sqrt{2}}[K^{+*}K^{-} + K^{-*}K^{+}]$

ASU

# $K^*K$ decays

- $K^+$ has $I = ½$ and $I_z = +½$ : $|\,½\,,+½\,\rangle$
- $K^-$ has $I = ½$ and $I_z = -½$ : $|\,½\,,-½\,\rangle$

For $I$=0 resonance $\rightarrow K^*K$ :

- $|\,0\,,0\,\rangle = \frac{1}{\sqrt{2}}[|\,½\,,+½\,\rangle|\,½\,,-½\,\rangle - |\,½\,,-½\,\rangle|\,½\,,+½\,\rangle]$
- $|\,0\,,0\,\rangle = \frac{1}{\sqrt{2}}[K^{+*}K^- - K^{-*}K^+]$

For $I$=1 resonance $\rightarrow K^*K$ :

- $|\,1\,,0\,\rangle = \frac{1}{\sqrt{2}}[|\,½\,,+½\,\rangle|\,½\,,-½\,\rangle + |\,½\,,-½\,\rangle|\,½\,,+½\,\rangle]$
- $|\,1\,,0\,\rangle = \frac{1}{\sqrt{2}}[K^{+*}K^- + K^{-*}K^+]$

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# Isospin

Meson resonances of interest:

- $\eta : I = 0$
- $f_1 : I = 0$
- $h_1 : I = 0$

# Included in the fit at each mass[$KK\pi$] bin

- Uniform background

# Included in the fit at each mass[$KK\pi$] bin

- Uniform background

- $J$=0, $m$= 0, $L$=0, $S$=0, Isobar = $a_0$
- $J$=0, $m$= 0, $L$=0, $S$=0, $KK\pi$
- $J$=0, $m$= 0, $L$=1, $S$=1, $KK\pi$
- $J$=0, $m$= 0, $L$=1, $S$=1, Isobar = $K^{*+}$
- $J$=0, $m$= 0, $L$=1, $S$=1, Isobar = $K^{*-}$

# Included in the fit at each mass[$KK\pi$] bin

- Uniform background

- $J$=0, $m$= 0, $L$=0, $S$=0, Isobar = $a_0$
- $J$=0, $m$= 0, $L$=0, $S$=0, $KK\pi$
- $J$=0, $m$= 0, $L$=1, $S$=1, $KK\pi$
- $J$=0, $m$= 0, $L$=1, $S$=1, Isobar = $K^{*+}$
- $J$=0, $m$= 0, $L$=1, $S$=1, Isobar = $K^{*-}$

- $J$=1, $m$=-1,0,1, $L$=1, $S$=0, Isobar = $a_0$
- $J$=1, $m$=-1,0,1, $L$=1, $S$=0, $KK\pi$
- $J$=1, $m$=-1,0,1, $L$=0, $S$=1, $KK\pi$
- $J$=1, $m$=-1,0,1, $L$=1, $S$=1, $KK\pi$
- $J$=1, $m$=-1,0,1, $L$=0, $S$=1, Isobar = $K^{*+}$
- $J$=1, $m$=-1,0,1, $L$=0, $S$=1, Isobar = $K^{*-}$

# AmpTools error bars

- Was using fit fractions to obtain fractions and errors, but did not know how to use fit fractions for subsets of amplitudes.

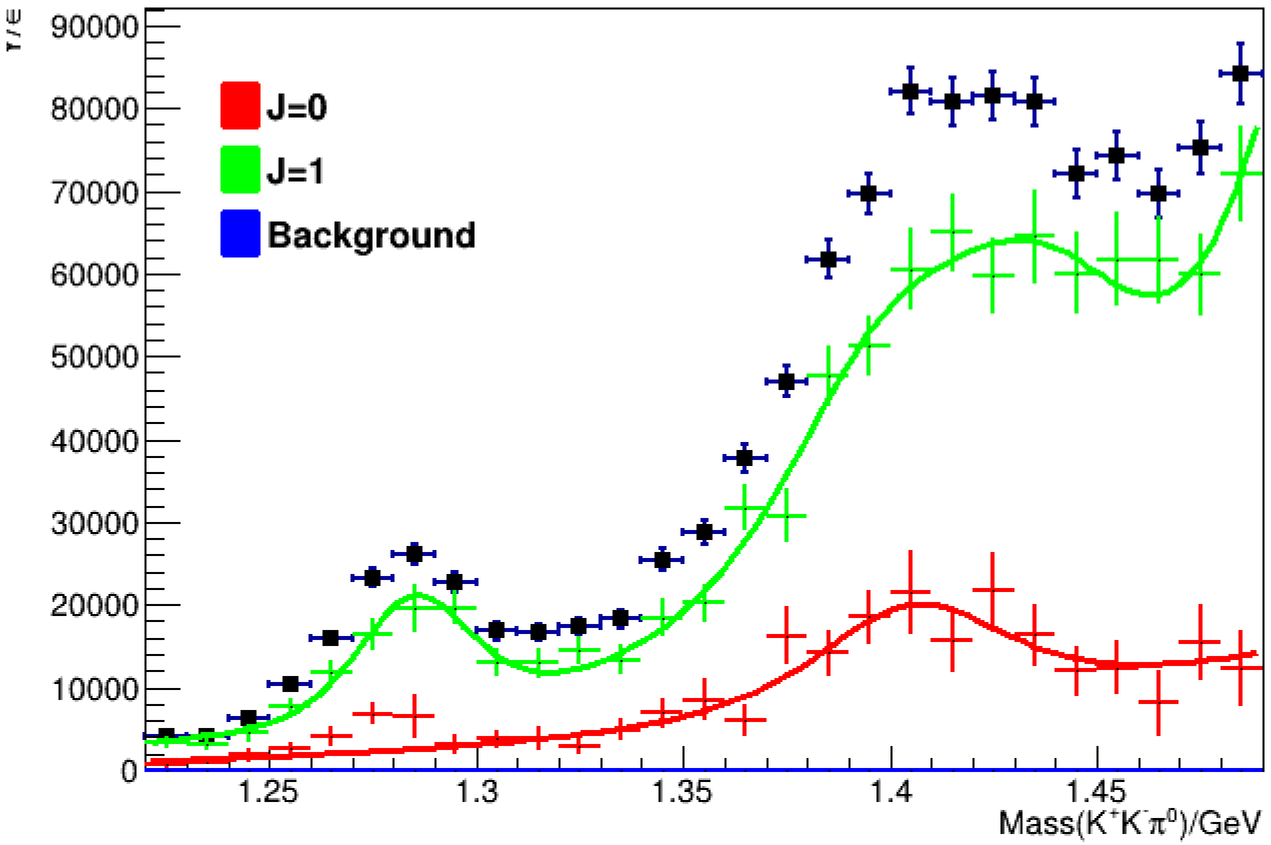
- Switched to using histograms created through plotGenerator.projection

Note:
Using plotGenerator.enableAmp(i) for the $i^{th}$ amplitude of interest in subset, and after turning on the desired amplitudes, creating histograms using Histogram* hist = plotGenerator.projection(ivar,reactionName,iplot), gives unrealistic error bars ☹

- For a few days, the error bars were destroyed and I used a temporary workaround ☹

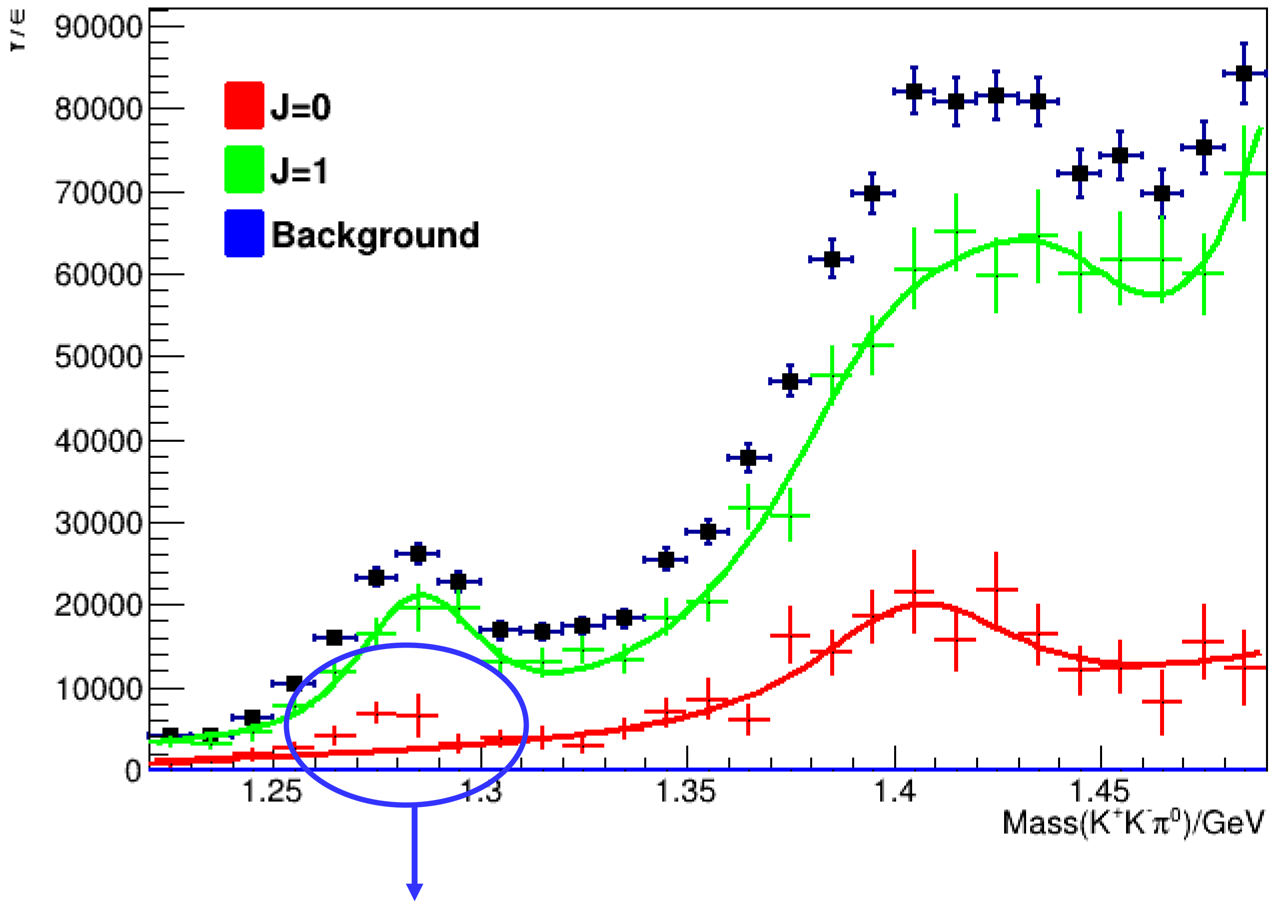
- Matt Shepard sent me instructions on how to get the fit fractions to work for a subset of amplitudes. Now error bars look good again ☺

# Fit



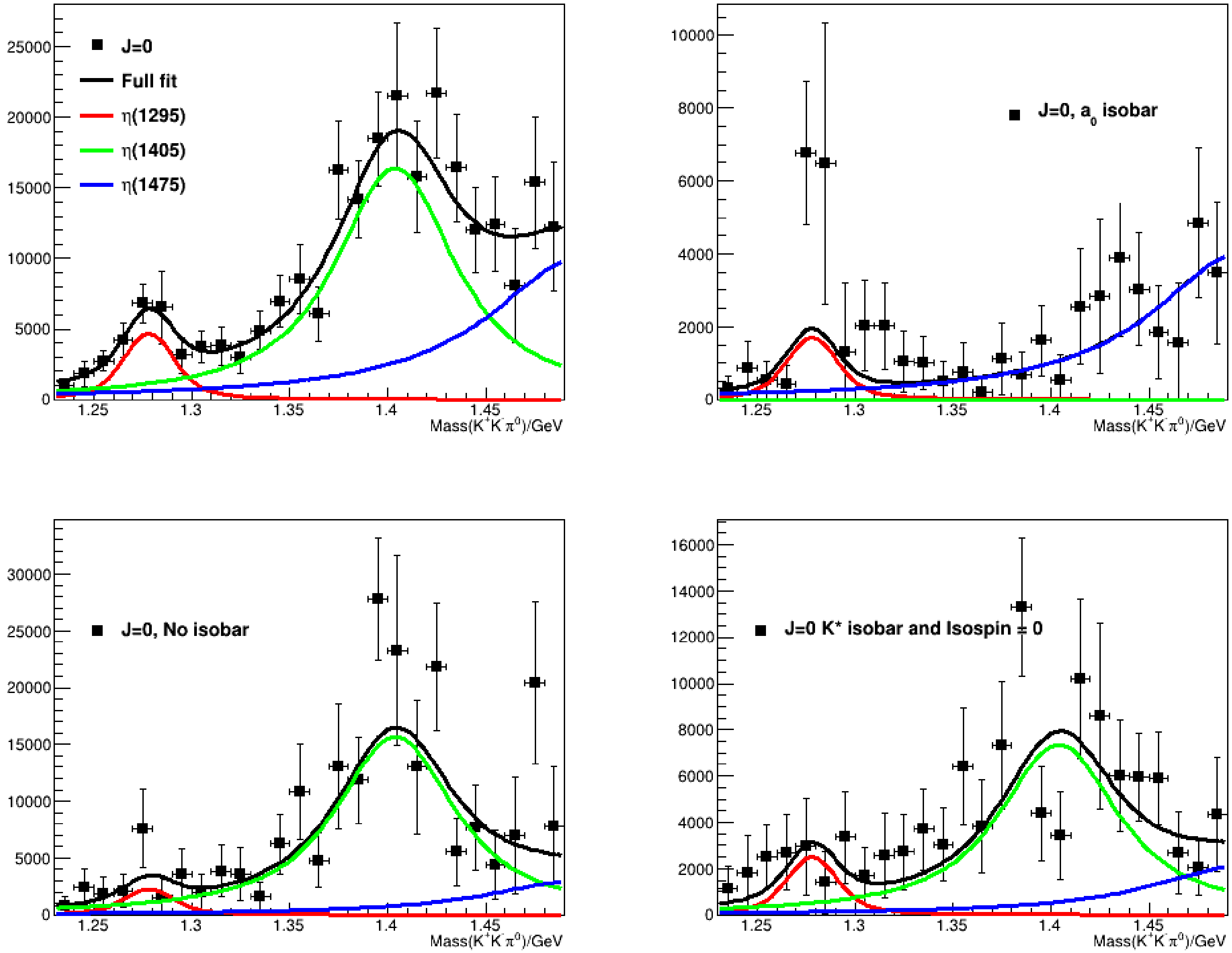
90000
80000
70000
60000
50000
40000
30000
20000
10000
0
J=0
J=1
Background
1.25
1.3
1.35
1.4
1.45
Mass(K⁺K⁻π⁰)/GeV

# Fit



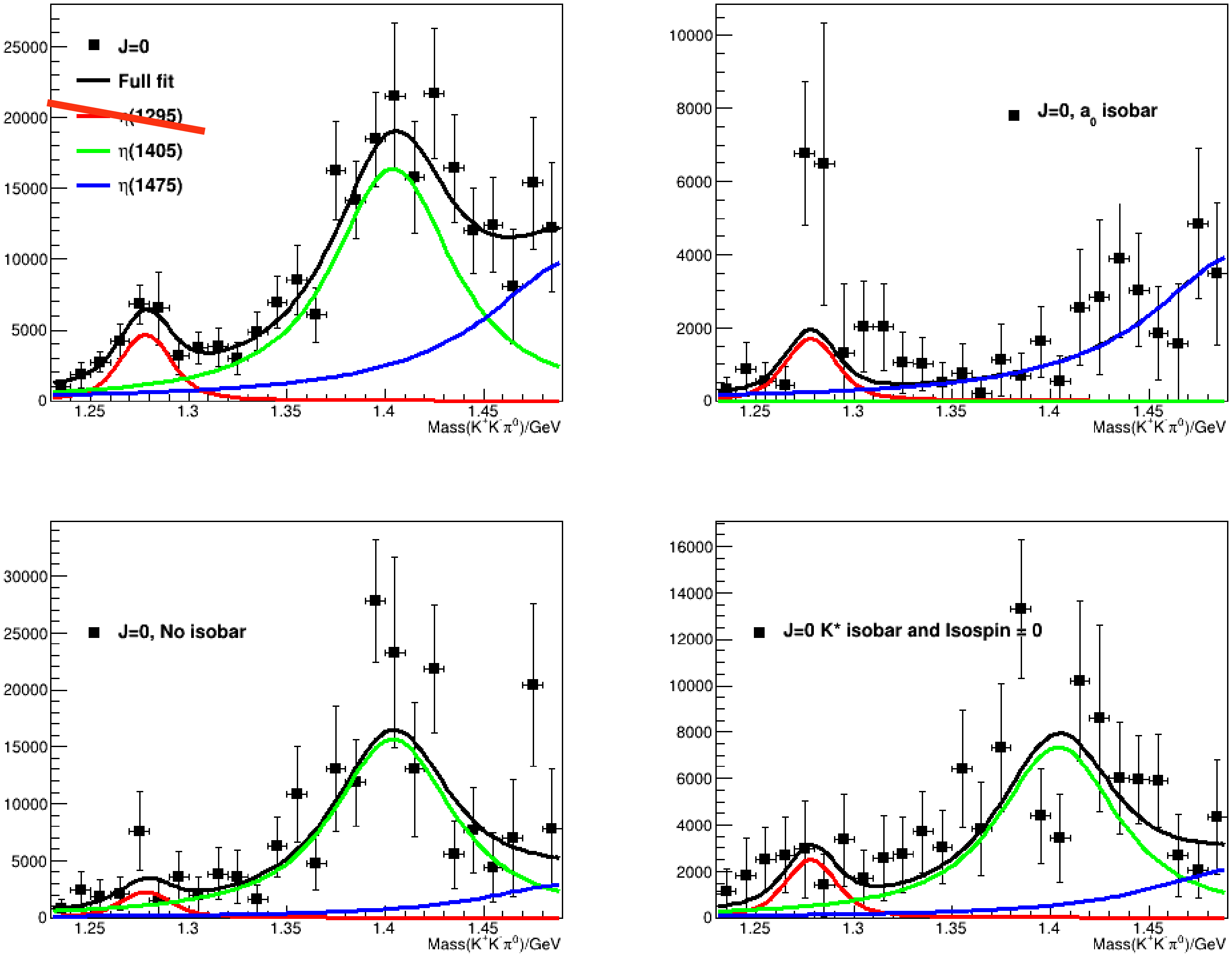

**Leakage from $J = 1$ ?**

# Simultaneous fit $J$=0



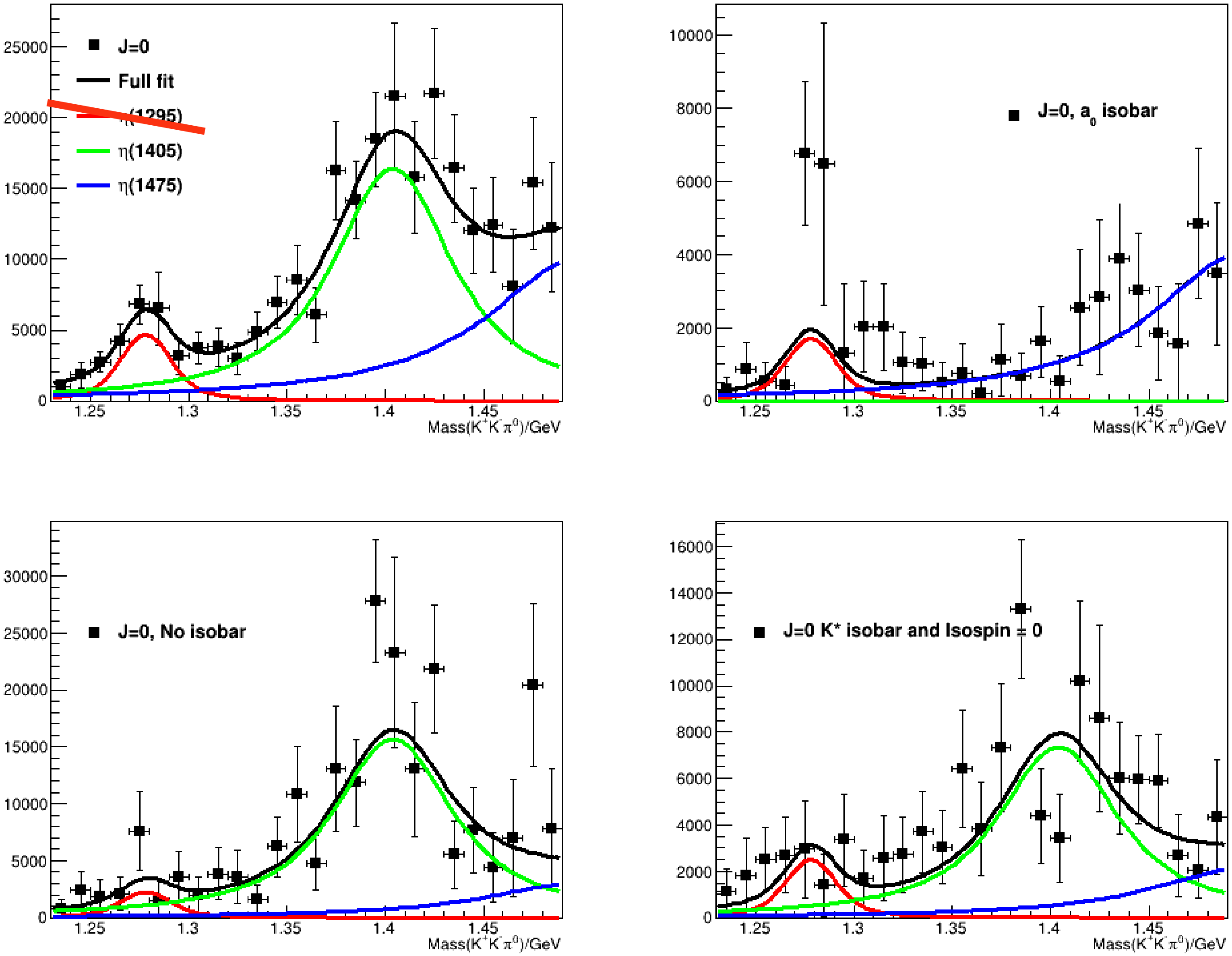

**Red bump**:
Center = 1278 +/- 4
Width = 15+/-11

# Simultaneous fit $J$=0




**Red bump**:
Center = 1278 +/- 4
Width = 15+/-11
**Consistent with leakage from *$f_1$*(1285)**

# Simultaneous fit $J$=0

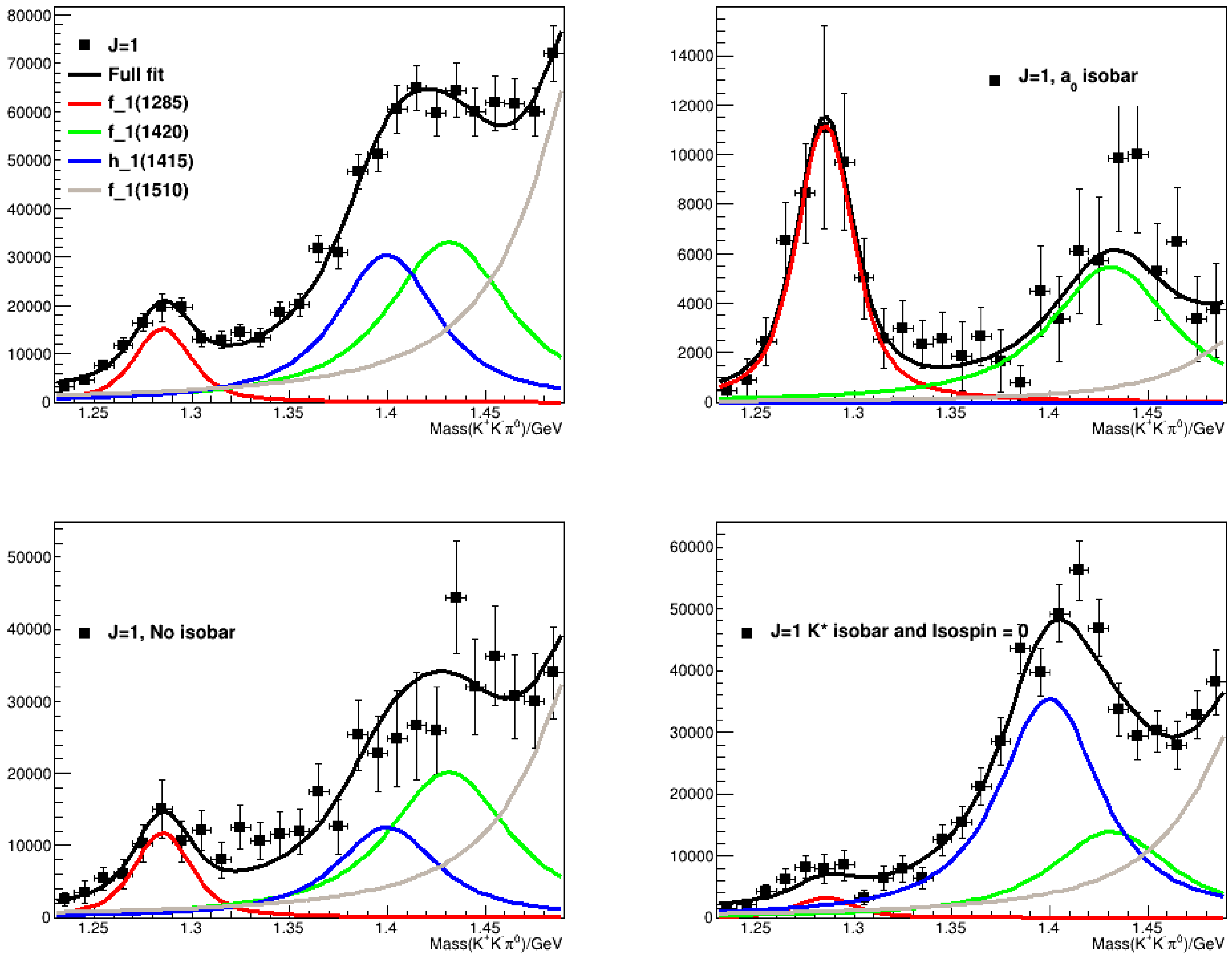



**Red bump**:
Center = 1278 +/- 4
Width = 15+/-11
**Consistent with leakage from $f_1$(1285)**

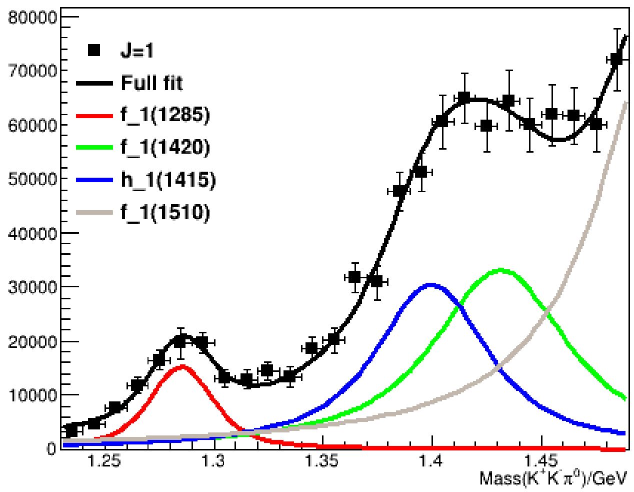
**Green** η(1405) :
Center = 1404 +/- 4
PDG = 1408 +/- 2

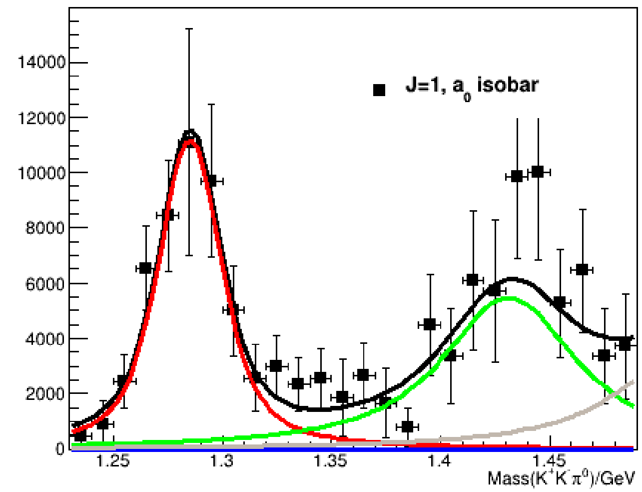
Width = 66 +/- 12
PDG = 50.1+/- 2.6

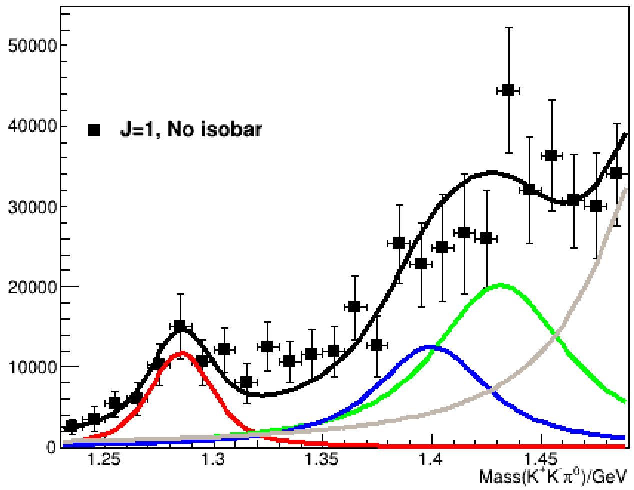
# Simultaneous fit *J*=1

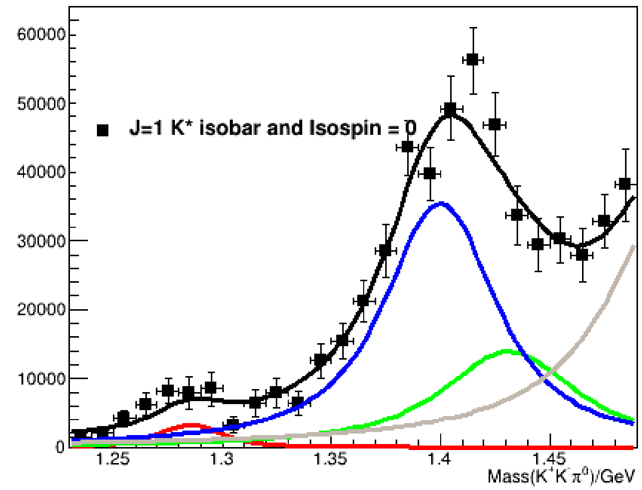


# Simultaneous fit *J*=1



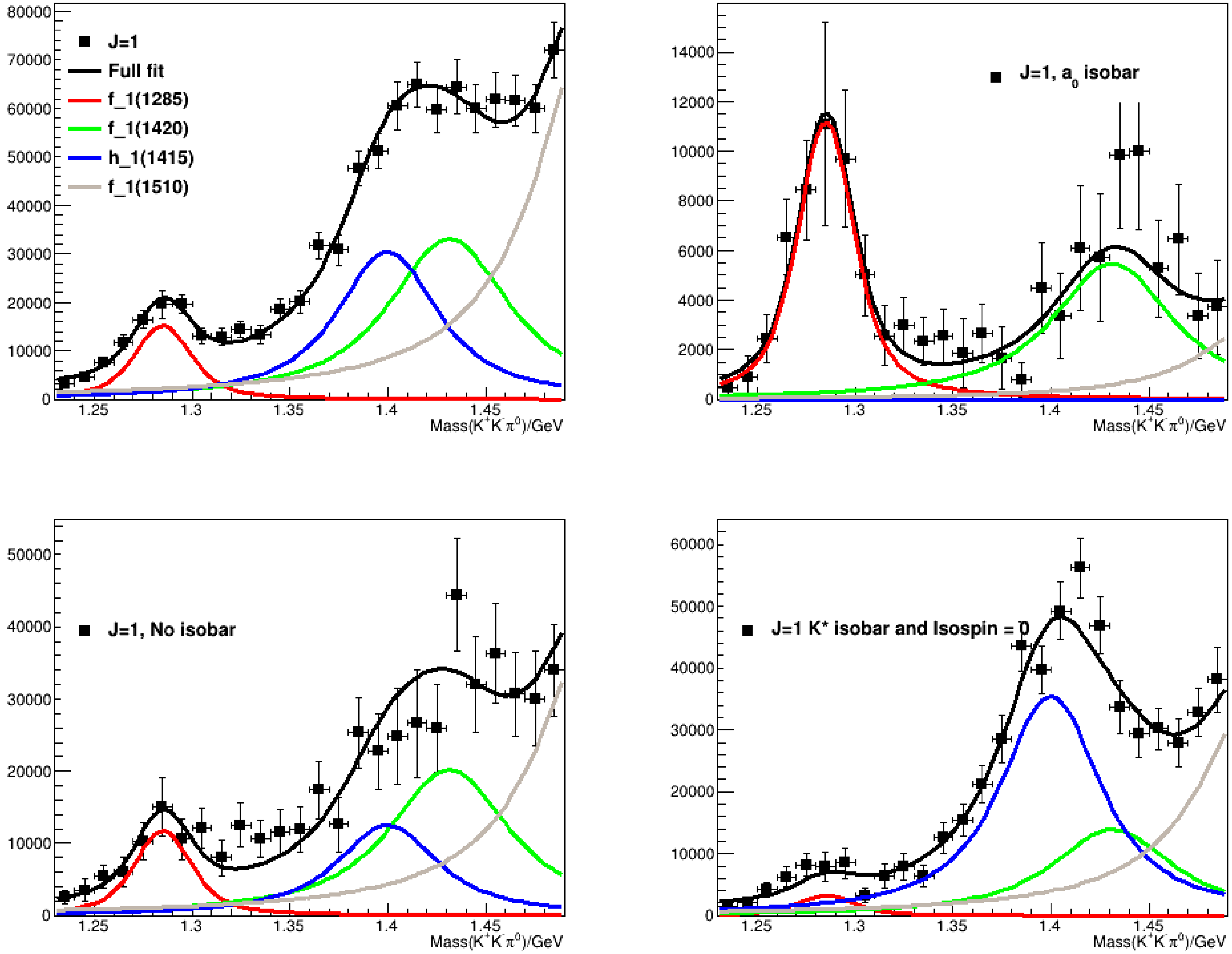




**Red** $f_1(1285)$:
Center = 1285 +/- 1
PDG = 1281.9 +/- 0.5

Width = 20 +/- 3
PDG = 22.7 +/- 1.1

# Simultaneous fit $J$=1



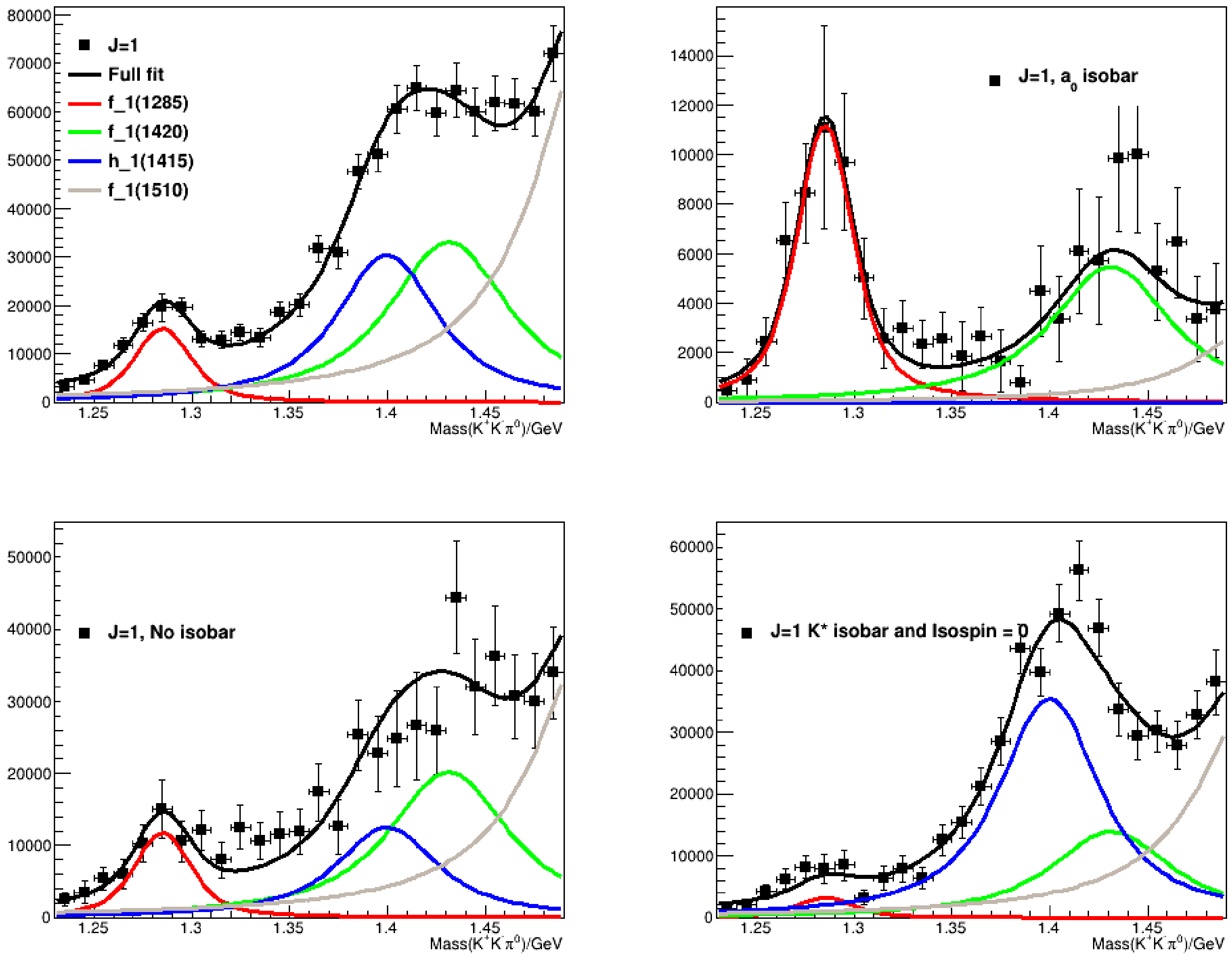

**Red** $f_1(1285)$:
Center = 1285 +/- 1
PDG = 1281.9 +/- 0.5

Width = 20 +/- 3
PDG = 22.7 +/- 1.1

**Green** $f_1(1420)$ :
Center = 1431 +/- 1
PDG = 1426.3 +/- 0.9

Width = 67 +/- 1
PDG = 54.5 +/- 2.6

# Simultaneous fit $J$=1




**Red** $f_1(1285)$:
Center = 1285 +/- 1
PDG = 1281.9 +/- 0.5

Width = 20 +/- 3
PDG = 22.7 +/- 1.1

**Green** $f_1(1420)$ :
Center = 1431 +/- 1
PDG = 1426.3 +/- 0.9

Width = 67 +/- 1
PDG = 54.5 +/- 2.6

**Blue** $h_1(1415)$ :
Center = 1400 +/- 4
PDG = 1409 + 9 – 8

Width = 54 +/- 6
PDG = 78 +/- 11

# $h_1(1415)$

**Blue** $h_1$(1415) :
Center = 1400 +/- 4
PDG = 1409 + 9 – 8

Width = 54 +/- 6
PDG = 78 +/- 11

# $h_1(1415)$

**Blue** $h_1(1415)$ :
Center = 1400 +/- 4
PDG = 1409 + 9 – 8

Width = 54 +/- 6
PDG = 78 +/- 11

Citation: R.L. Workman *et al.* (Particle Data Group), Prog.Theor.Exp.Phys. **2022**, 083C01 (2022) and 2023 update

## $h_1(1415)$

$I^G(J^{PC}) = 0^-(1^{+-})$

was $h_1(1380)$

### $h_1(1415)$ MASS

| VALUE (MeV) | EVTS | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|---|
| **$1409^{+9}_{-8}$ OUR AVERAGE** | | Error includes scale factor of 1.9. See the ideogram below. | | | |
| $1384 \pm 6\ ^{+9}_{-0}$ | | [1] ABLIKIM | 22C | BES3 | $J/\psi \to \gamma\eta'\eta' \to 4/5\gamma 2(\pi^+\pi^-)$ |
| $1423 \pm 2.1 \pm 7.3$ | 2.2k | [2] ABLIKIM | 18AB | BES3 | $J/\psi \to \eta' h_1 \to \eta' K^* \overline{K}$ |
| $1412 \pm 4\ \pm 8$ | | [2] ABLIKIM | 15M | BES3 | $\psi(2S) \to \gamma\chi_{c1,2} \to \gamma\phi(h_1 \to K^*\overline{K})$ |
| $1440 \pm 60$ | | ABELE | 97H | CBAR | $\overline{p}p \to K^0_L K^0_S \pi^0\pi^0$ |
| $1380 \pm 20$ | | ASTON | 88C | LASS | $11\ K^- p \to K^0_S K^\pm \pi^\mp \Lambda$ |

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# $h_1(1415)$

**Blue** $h_1(1415)$ :
Center = 1400 +/- 4
PDG = 1409 + 9 – 8

Width = 54 +/- 6
PDG = 78 +/- 11

Citation: R.L. Workman *et al.* (Particle Data Group), Prog.Theor.Exp.Phys. **2022**, 083C01 (2022) and 2023 update

**$h_1(1415)$** $I^G(J^{PC}) = 0^-(1^{+-})$

was $h_1(1380)$

## $h_1(1415)$ MASS

| VALUE (MeV) | EVTS | DOCUMENT ID | TECN | COMMENT |
|---|---|---|---|---|
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| $1384 \pm 6\ ^{+9}_{-0}$ | | [1] ABLIKIM | 22C BES3 | $J/\psi \to \gamma\eta'\eta' \to 4/5\gamma 2(\pi^+\pi^-)$ |
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| $1440 \pm 60$ | | ABELE | 97H CBAR | $\overline{p}p \to K^0_L K^0_S \pi^0\pi^0$ |
| $1380 \pm 20$ | | ASTON | 88C LASS | $11\ K^- p \to K^0_S K^\pm \pi^\mp \Lambda$ |

## $h_1(1415)$ WIDTH

| VALUE (MeV) | EVTS | DOCUMENT ID | TECN | COMMENT |
|---|---|---|---|---|
| **$78 \pm 11$ OUR AVERAGE** | | | | |
| $66 \pm 10\ ^{+12}_{-10}$ | | [1] ABLIKIM | 22C BES3 | $J/\psi \to \gamma\eta'\eta' \to 4/5\gamma 2(\pi^+\pi^-)$ |
| $90.3 \pm 9.8 \pm 17.5$ | 2.2k | [2] ABLIKIM | 18AB BES3 | $J/\psi \to \eta' h_1 \to \eta' K^* \overline{K}$ |
| $84 \pm 12 \pm 40$ | | [2] ABLIKIM | 15M BES3 | $\psi(2S) \to \gamma\chi_{c1,2} \to \gamma\phi(h_1 \to K^*\overline{K})$ |
| $170 \pm 80$ | | ABELE | 97H CBAR | $\overline{p}p \to K^0_L K^0_S \pi^0\pi^0$ |
| $80 \pm 30$ | | ASTON | 88C LASS | $11\ K^- p \to K^0_S K^\pm \pi^\mp \Lambda$ |

[1] From a partial wave analysis of the systems $(\gamma X)$, with $X \to \eta'\eta'$, and $(\eta' X)$, with $X \to \gamma\eta'$ in the decay $J/\psi \to \gamma\eta'\eta'$. The intermediate resonance $X$ is parametrized by a constant-width, relativistic Breit-Wigner.
[2] Final states $K^+K^-\pi^0$ and $K^0_S K^\pm \pi^\mp$.

## $h_1(1415)$ DECAY MODES

| | Mode |
|---|---|
| $\Gamma_1$ | $K\overline{K}^*(892)$+ c.c. |

## $h_1(1415)$ REFERENCES


| | | | | |
|---|---|---|---|---|
| ABLIKIM | 22C | PR D105 072002 | M. Ablikim *et al.* | (BESIII Collab.) |
| ABLIKIM | 18AB | PR D98 072005 | M. Ablikim *et al.* | (BESIII Collab.) |
| ABLIKIM | 15M | PR D91 112008 | M. Ablikim *et al.* | (BESIII Collab.) |
| ABELE | 97H | PL B415 280 | A. Abele *et al.* | (Crystal Barrel Collab.) |
| ASTON | 88C | PL B201 573 | D. Aston *et al.* | (SLAC, NAGO, CINC, INUS) |


- **Width consistent with most recent measurement**

# Title