Today Review

$\boxed{L20}$

Angular velocity ≡ eee

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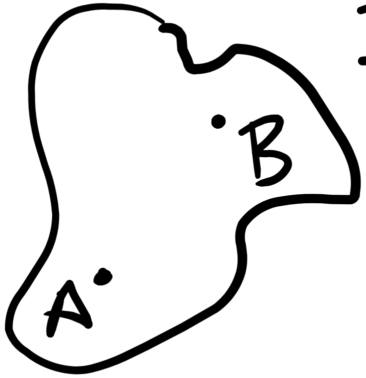
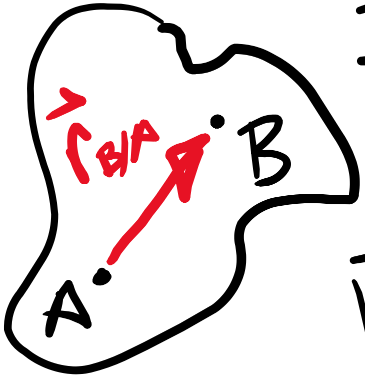
The acceleration can be written as

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times [\vec{\omega} \times \vec{r}]$$



If points A & B are on a rigid body in plane motion



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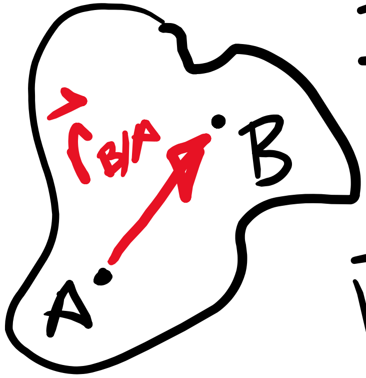

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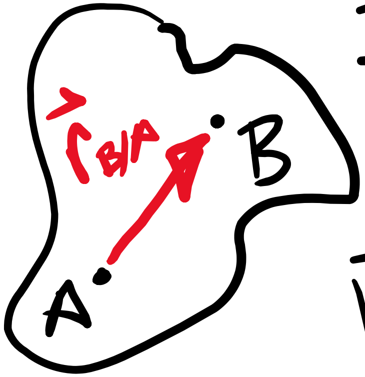

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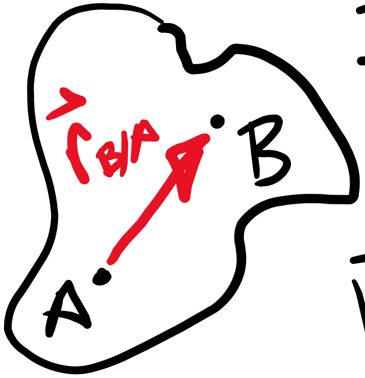
Rotating disk



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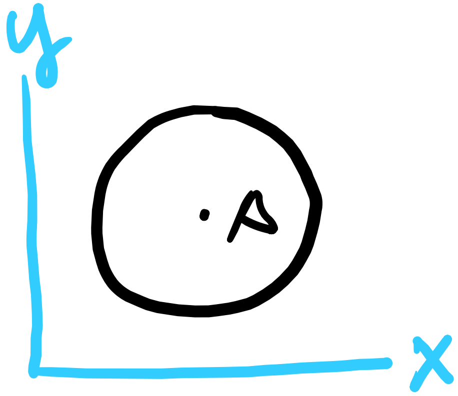

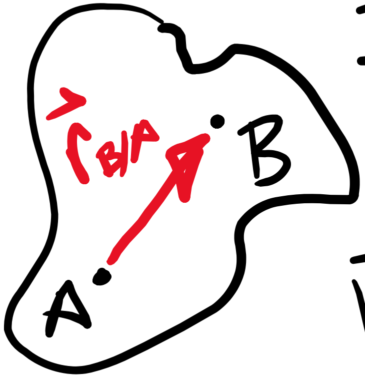

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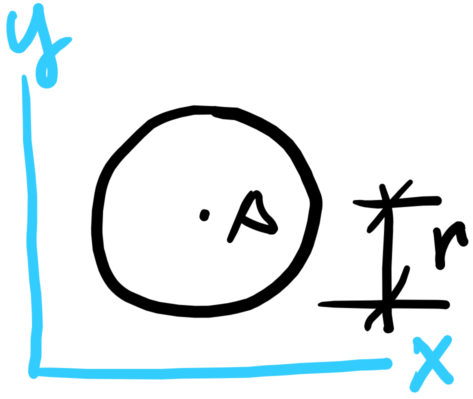

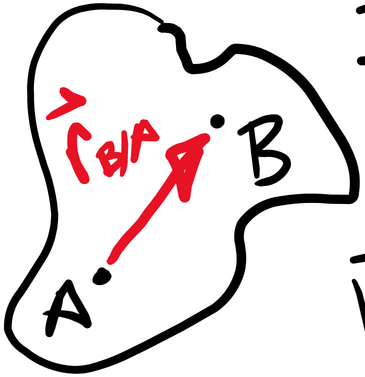

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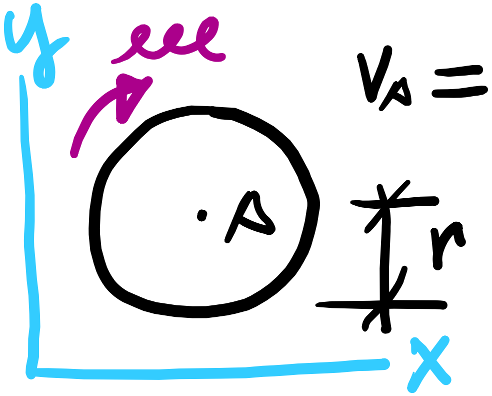
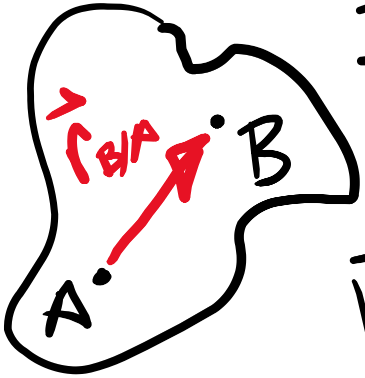


$V_A = 0$



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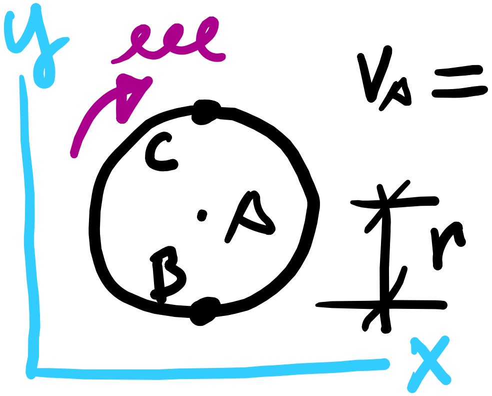

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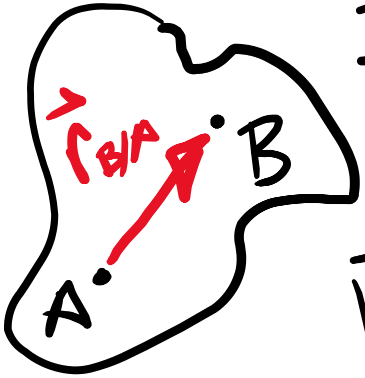

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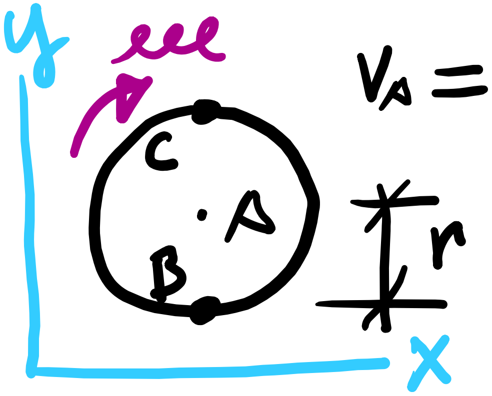
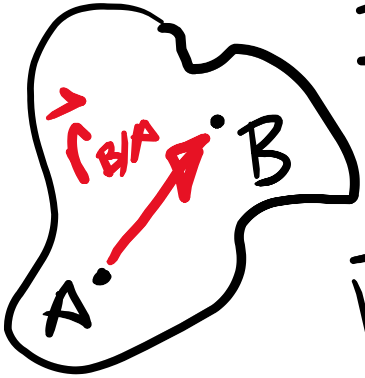


$V_A = 0$

$\Rightarrow \vec{V}_{B/A} = r\omega(-\hat{x})$



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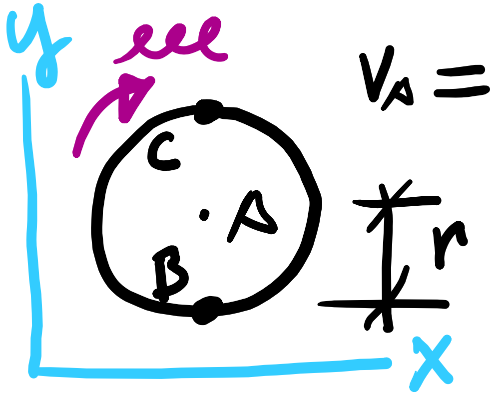

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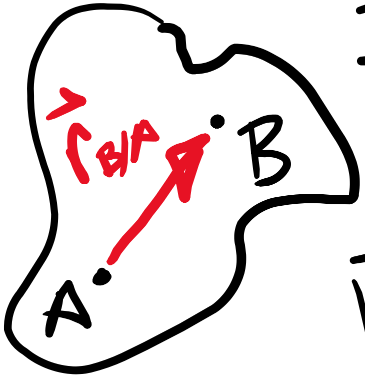

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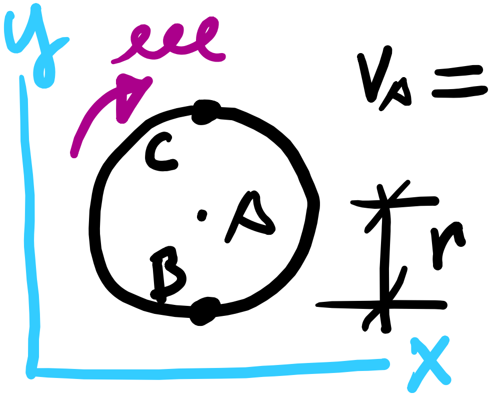

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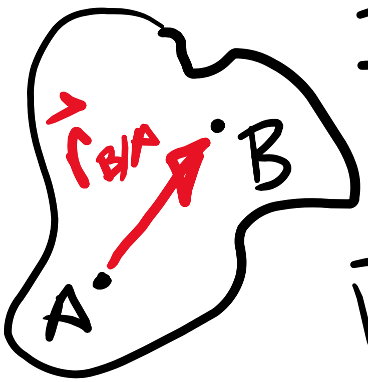

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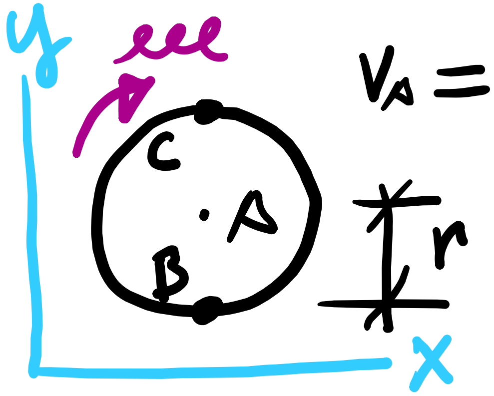

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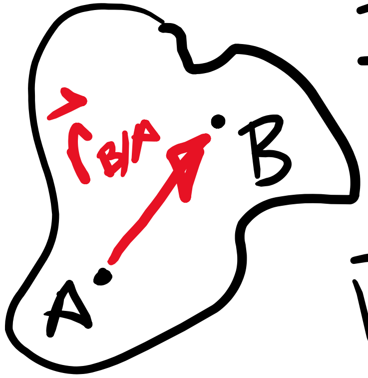

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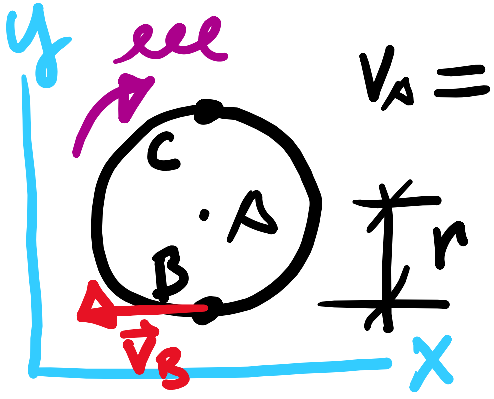

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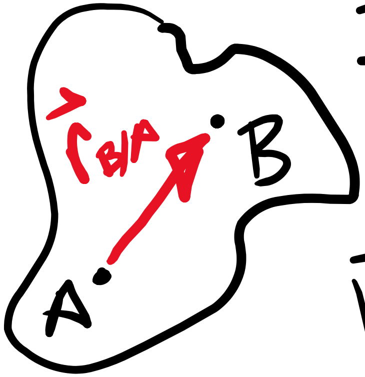

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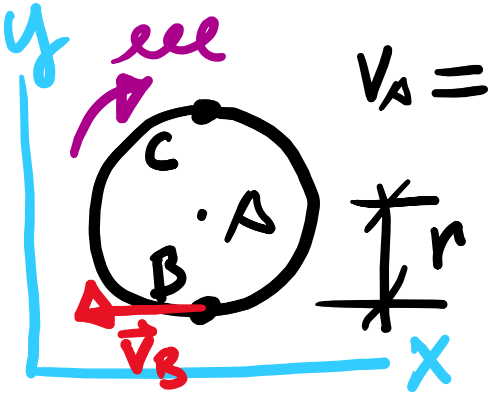

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If disk is rolling without slipping

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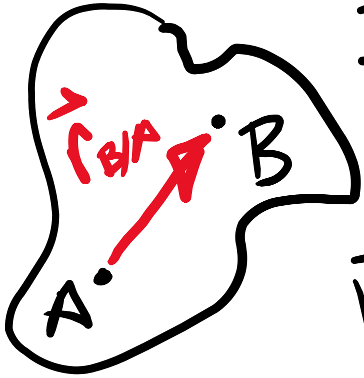

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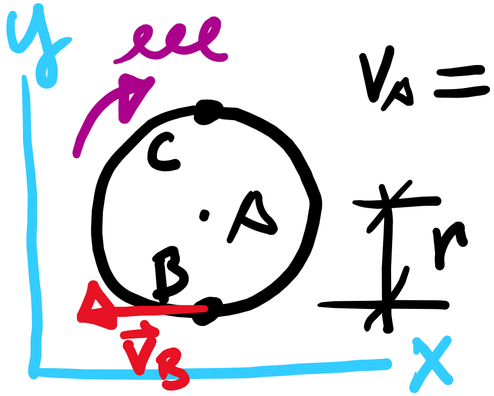
Rotating disk



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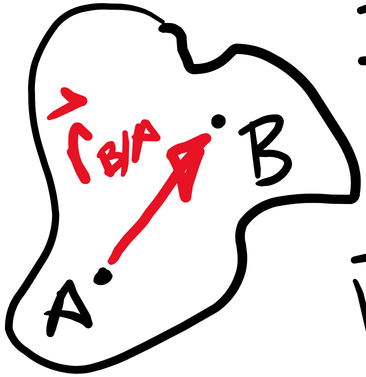
If disk is rolling without slipping

$\vec{V}_A = r\omega\hat{x}$ & we still have $\vec{V}_{B/A} = r\omega(-\hat{x})$



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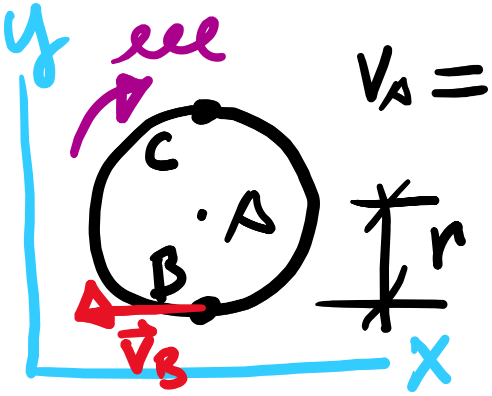
Rotating disk



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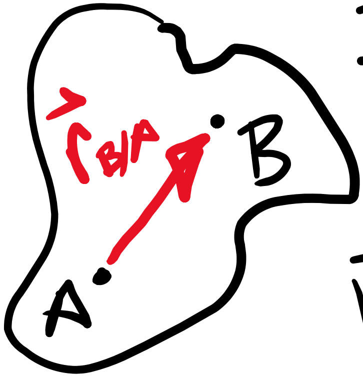
$\vec{V}_A = r\omega\hat{x}$ & we still have $\vec{V}_{B/A} = r\omega(-\hat{x})$

And this gives us $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$



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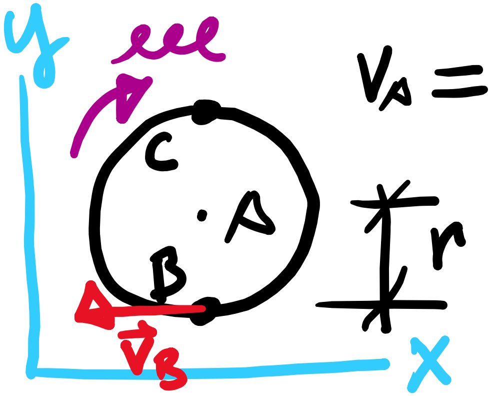
Rotating disk



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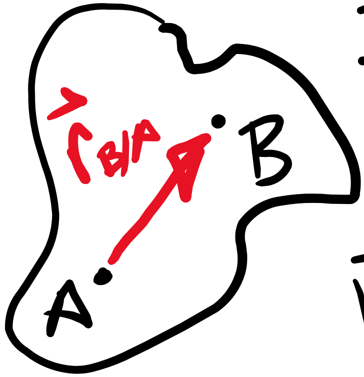
If disk is rolling without slipping

$\vec{V}_A = r\omega\hat{x}$ & we still have $\vec{V}_{B/A} = r\omega(-\hat{x})$

And this gives us $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A = r\omega\hat{x} - r\omega\hat{x}$



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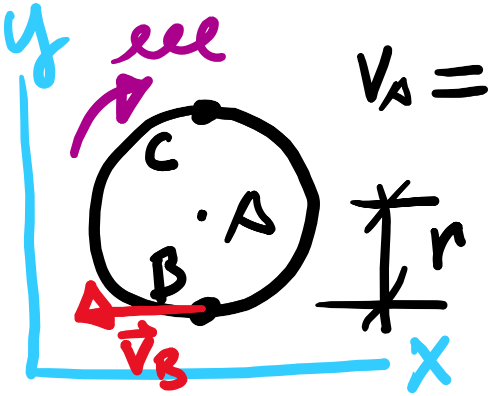
Rotating disk



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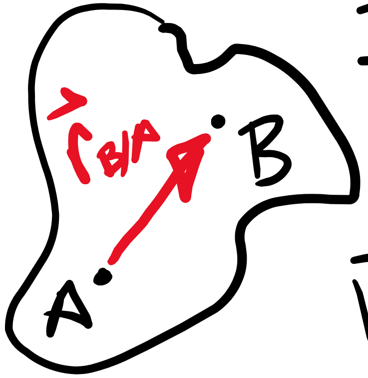
If disk is rolling without slipping

$\vec{V}_A = r\omega\hat{x}$ & we still have $\vec{V}_{B/A} = r\omega(-\hat{x})$

And this gives us $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A = r\omega\hat{x} - r\omega\hat{x} = 0$



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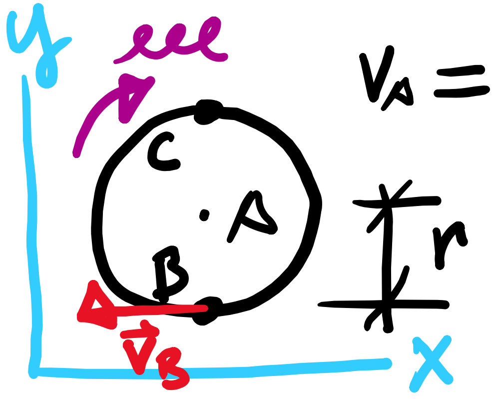
Rotating disk



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$\Rightarrow \vec{V}_B = \vec{V}_{B/A} + \overset{\cancel{0}}{\vec{V}_A} \Rightarrow$

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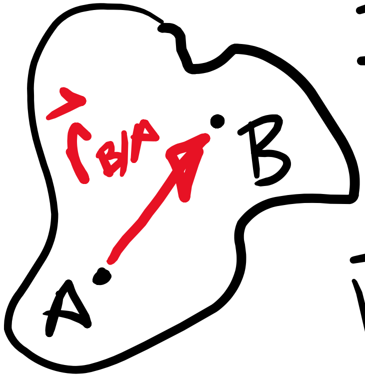
If disk is rolling without slipping

$\vec{V}_A = r\omega\hat{x}$ & we still have $\vec{V}_{B/A} = r\omega(-\hat{x})$

And this gives us $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A = r\omega\hat{x} - r\omega\hat{x} = 0$

So $\vec{V}_B$ is at rest at this moment in time!



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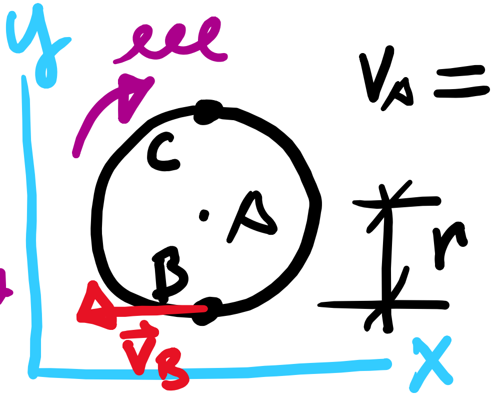
$$\vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{V}_{B/A}| = \omega r_{B/A}$$

Rotating disk

What about point C at top of disk?



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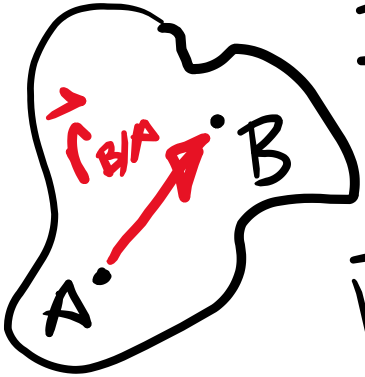

If points A & B are on a rigid body in plane motion

$$\vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{V}_{B/A}| = \omega r_{B/A}$$

Rotating disk

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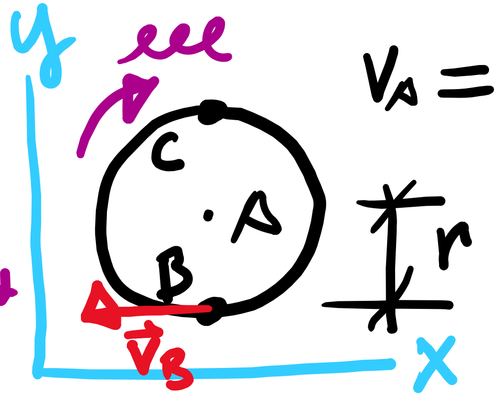

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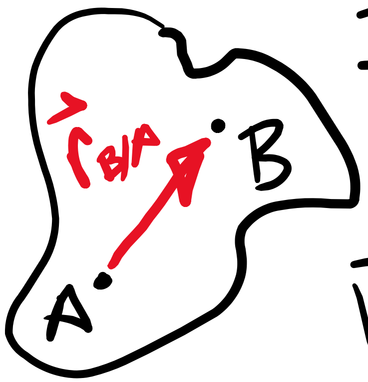
$\Rightarrow \vec{V}_{B/A} = r\omega(-\hat{x})$

$\Rightarrow \vec{V}_B = \vec{V}_{B/A} + \overset{0}{\cancel{\vec{V}_A}} \Rightarrow$

$\vec{V}_B = r\omega(-\hat{x})$

If disk is rolling without slipping

$$V_c = \vec{V}_{C/A} + \vec{V}_A$$



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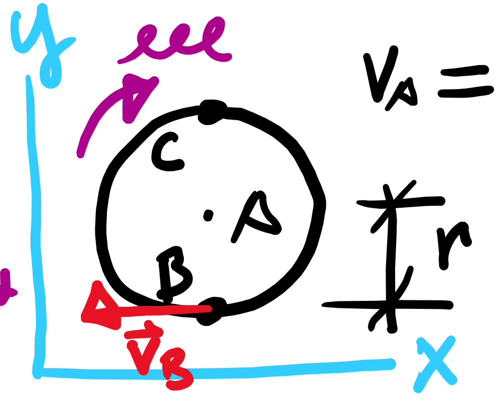


$V_A = 0$

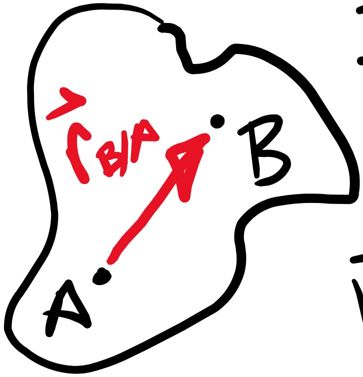
$\Rightarrow \vec{V}_{B/A} = r\omega(-\hat{x})$

$\Rightarrow \vec{V}_B = \vec{V}_{B/A} + \cancel{\vec{V}_A}^{\,0} \Rightarrow$

$\vec{V}_B = r\omega(-\hat{x})$

If disk is rolling without slipping

$V_c = \vec{V}_{c/A} + \vec{V}_A$, but $\vec{V}_{c/A} = r\omega\hat{x}$

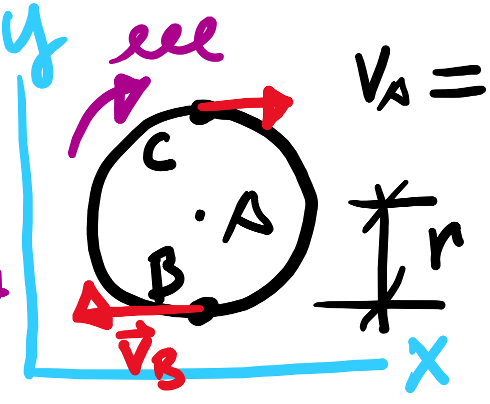


If points A & B are on a rigid body in plane motion

$$\vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{V}_{B/A}| = \omega r_{B/A}$$

Rotating disk

What about point C at top of disk?



$V_A = 0$

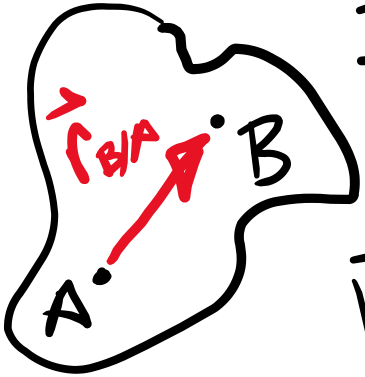
$\Rightarrow \vec{V}_{B/A} = r\omega(-\hat{x})$

$\Rightarrow \vec{V}_B = \vec{V}_{B/A} + \cancelto{0}{\vec{V}_A} \Rightarrow$

$\vec{V}_B = r\omega(-\hat{x})$

If disk is rolling without slipping

$V_c = \vec{V}_{c/A} + \vec{V}_A$, but $\vec{V}_{c/A} = r\omega\hat{x}$

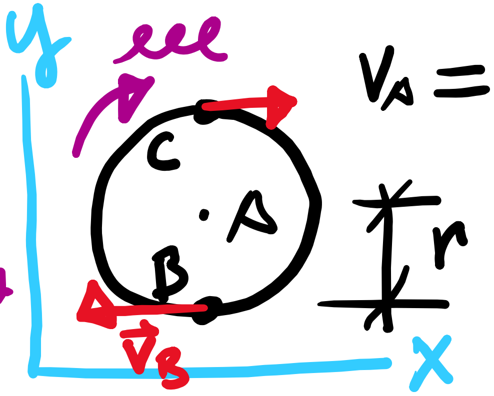


If points A & B are on a rigid body in plane motion

$$\vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{V}_{B/A}| = \omega r_{B/A}$$

Rotating disk

What about point C at top of disk?



$V_A = 0$

$\Rightarrow \vec{V}_{B/A} = r\omega(-\hat{x})$

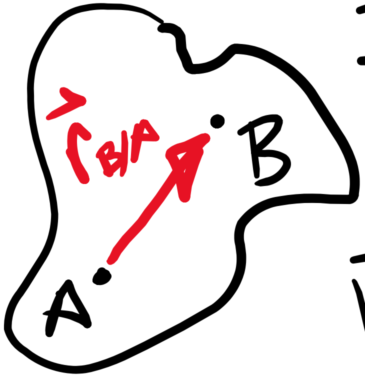
$\Rightarrow \vec{V}_B = \vec{V}_{B/A} + \cancelto{0}{\vec{V}_A} \Rightarrow$

$\vec{V}_B = r\omega(-\hat{x})$

If disk is rolling without slipping

$\vec{V}_C = \vec{V}_{C/A} + \vec{V}_A$, but $\vec{V}_{C/A} = r\omega\hat{x}$ So

$\vec{V}_C = r\omega\hat{x} + r\omega\hat{x}$

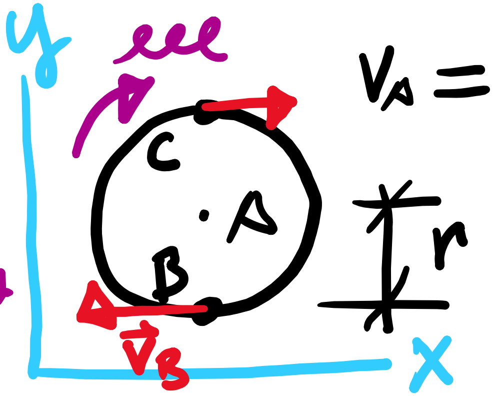


If points A & B are on a rigid body in plane motion

$$\vec V_{B/A} = \vec\omega \times \vec r_{B/A} \Rightarrow |\vec V_{B/A}| = \omega r_{B/A}$$

Rotating disk

What about point C at top of disk?



$V_A = 0$

$\Rightarrow \vec V_{B/A} = r\omega(-\hat x)$

$\Rightarrow \vec V_B = \vec V_{B/A} + \cancelto{0}{\vec V_A} \Rightarrow$

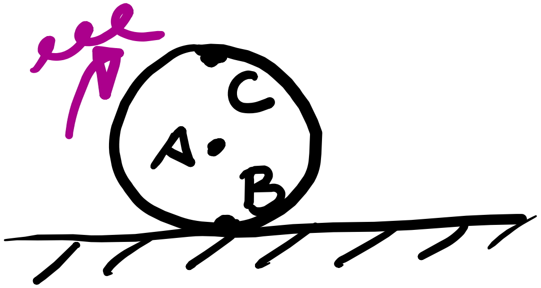
$\vec V_B = r\omega(-\hat x)$

If disk is rolling without slipping

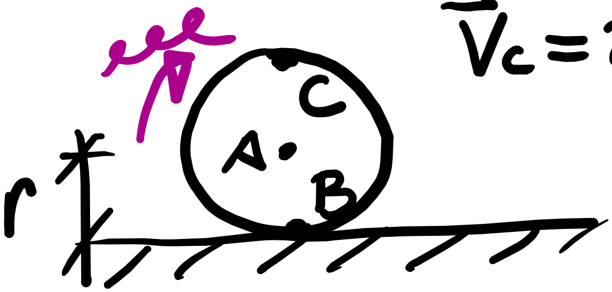
$\vec V_C = \vec V_{C/A} + \vec V_A$, but $\vec V_{C/A} = r\omega\hat x$ So

$\vec V_C = r\omega\hat x + r\omega\hat x \Rightarrow \vec V_C = 2r\omega\hat x$

Wheel rolling with no slipping

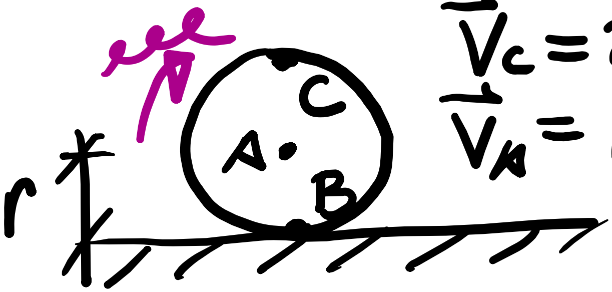


Wheel rolling with no slipping



$\vec{V}_c = 2r\,eee\,\hat{x}$

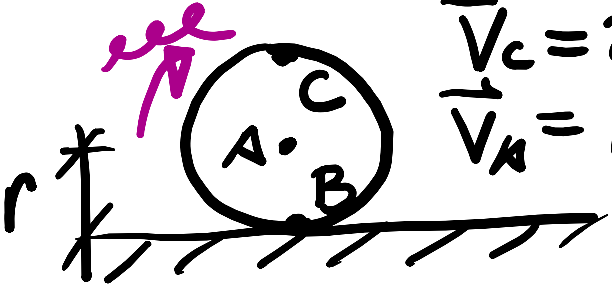
Wheel rolling with no slipping



$\vec{V}_C = 2r\omega\,\hat{x}$

$\vec{V}_A = r\omega\,\hat{x}$

Wheel rolling with no slipping

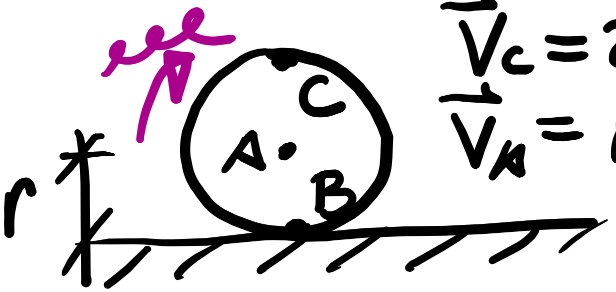


$\vec{V}_C = 2r\omega\omega\omega\,\hat{x}$

$\vec{V}_A = r\omega\omega\omega\,\hat{x}$

$\&\ \vec{V}_B = \alpha$

Wheel rolling with no slipping



$\vec{V}_C = 2r\omega\hat{x}$

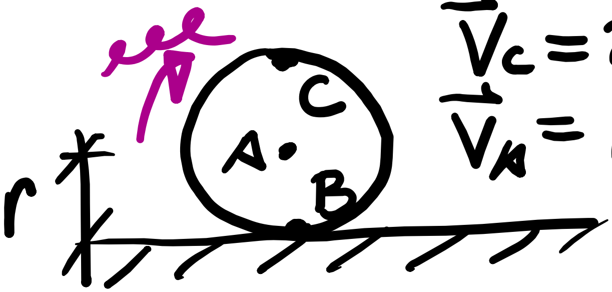
$\vec{V}_A = r\omega\hat{x}$

& $\vec{V}_B = \alpha$

We can now write

$\vec{V}_{C/B} = \vec{V}_C - \vec{V}_B$

Wheel rolling with no slipping



$\vec{V}_C = 2r\omega\hat{x}$

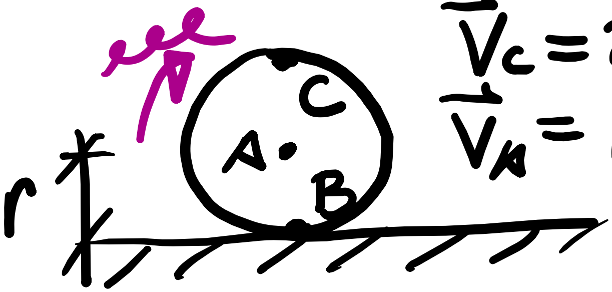
$\vec{V}_A = r\omega\hat{x}$

& $\vec{V}_B = 0$

We can now write

$$\vec{V}_{C/B} = \vec{V}_C - \vec{V}_B = 2r\omega\hat{x}$$

Wheel rolling with no slipping



$\vec{V}_C = 2r\omega\hat{x}$

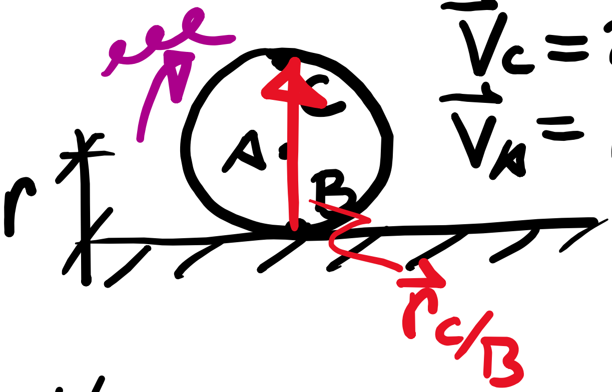
$\vec{V}_A = r\omega\hat{x}$

& $\vec{V}_B = 0$

We can now write $\vec{V}_{C/B} = \vec{V}_C - \vec{V}_B = 2r\omega\hat{x}$. However, we could have just written this down using

$V_{C/B} = r_{C/B}\omega$

Wheel rolling with no slipping



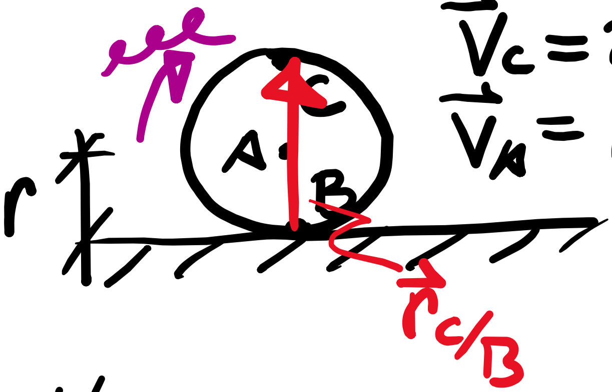
$\vec{V}_C = 2r\omega\hat{x}$

$\vec{V}_A = r\omega\hat{x}$

& $\vec{V}_B = 0$

We can now write $\vec{V}_{C/B} = \vec{V}_C - \vec{V}_B = 2r\omega\hat{x}$. However, we could have just written this down using $V_{C/B} = r_{C/B}\omega$

Wheel rolling with no slipping



$\vec{V}_c = 2r\omega\hat{x}$

$\vec{V}_A = r\omega\hat{x}$

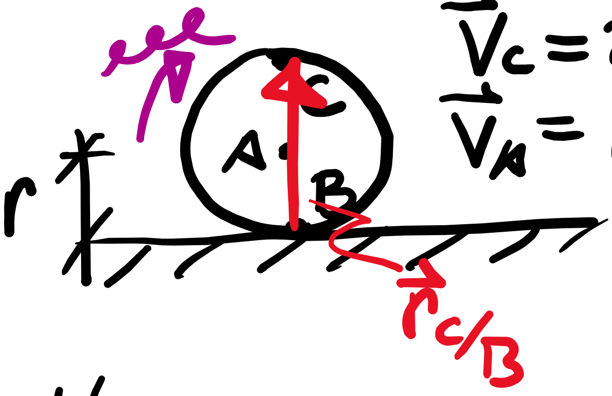
& $\vec{V}_B = 0$

We can now write

$\vec{V}_{c/B} = \vec{V}_c - \vec{V}_B = 2r\omega\hat{x}$. However, we could have just written this down using

$V_{c/B} = r_{c/B}\omega = 2r\omega$

Wheel rolling with no slipping



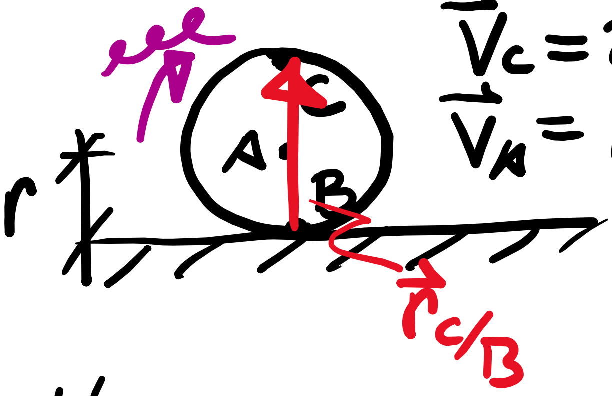
$\vec V_C = 2r\omega\hat x$

$\vec V_A = r\omega\hat x$

& $\vec V_B = 0$

We can now write $\vec V_{c/B} = \vec V_c - \vec V_B = 2r\omega\hat x$. However, we could have just written this down using $V_{c/B} = r_{c/B}\omega = 2r\omega$, since "The angular velocity is independent of the reference point"

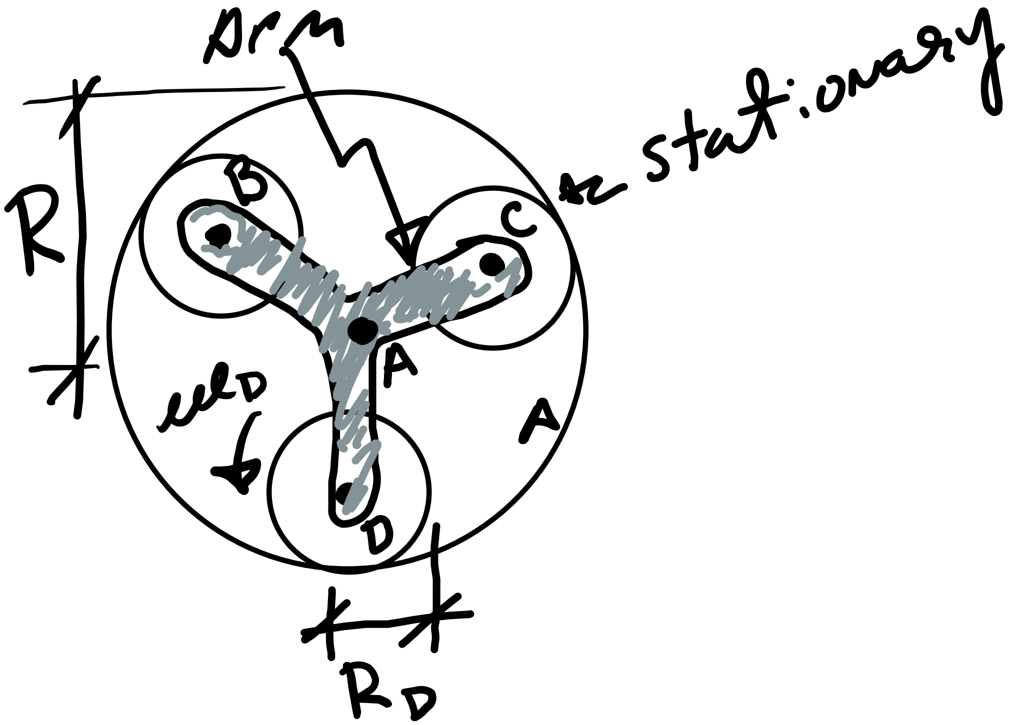
Wheel rolling with no slipping

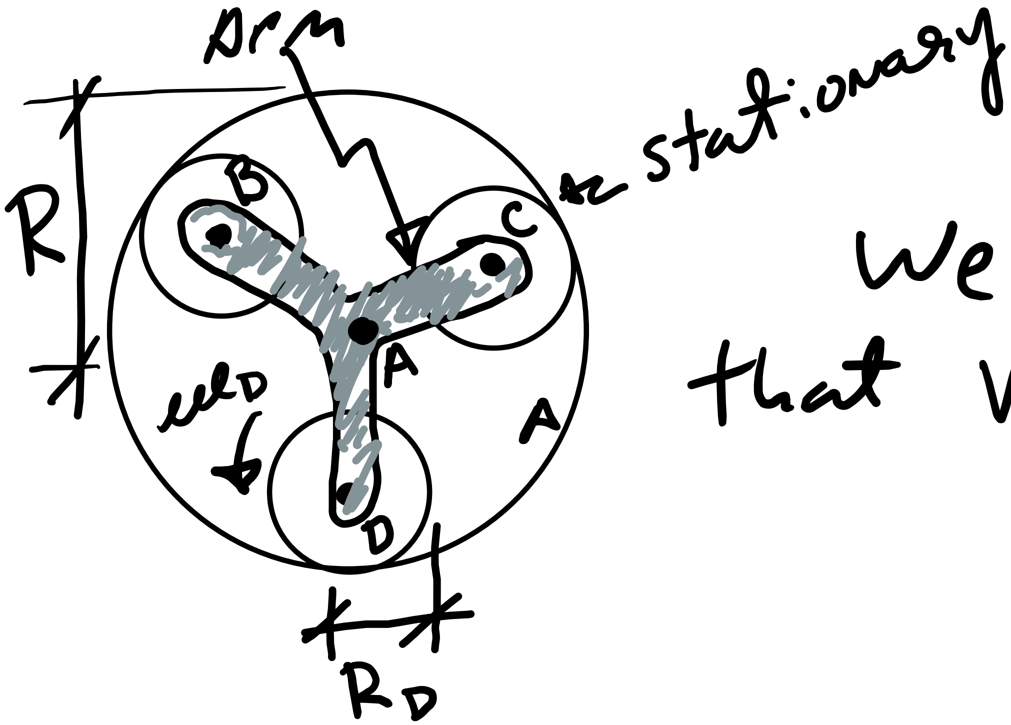


$\vec{V}_C = 2r\omega\hat{x}$
$\vec{V}_A = r\omega\hat{x}$
& $\vec{V}_B = 0$

We can now write
$\vec{V}_{C/B} = \vec{V}_C - \vec{V}_B = 2r\omega\hat{x}$. However, we could have just written this down using $V_{C/B} = r_{C/B}\omega = 2r\omega$, since "The angular velocity is independent of the reference point" & $r_{C/B} = 2r$

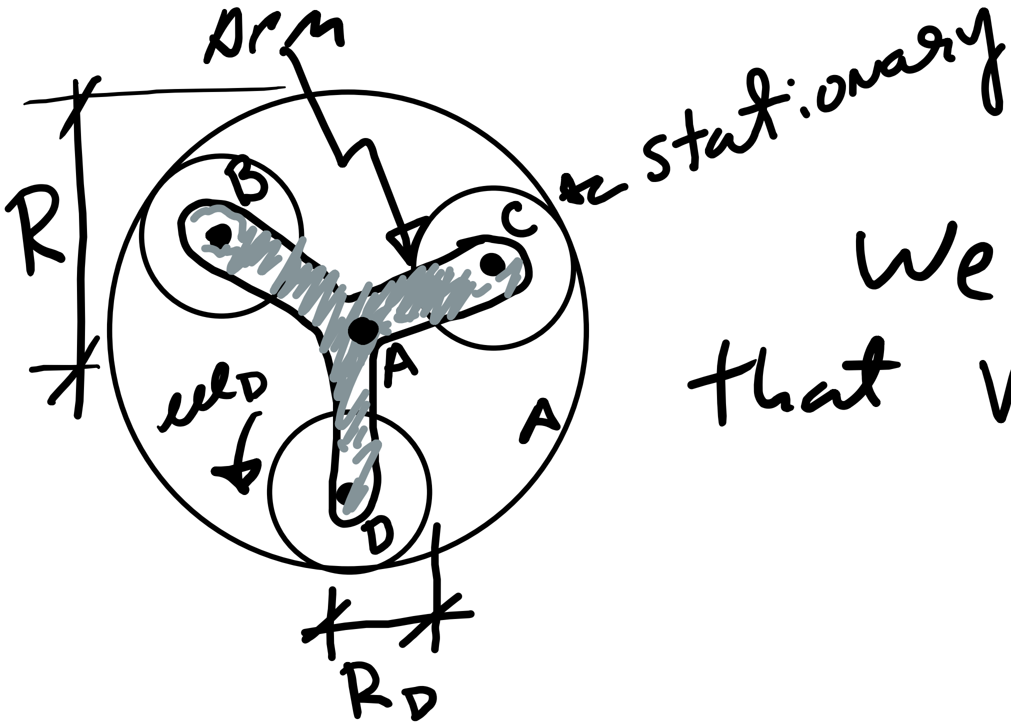
Example: Find ω_{ARM}





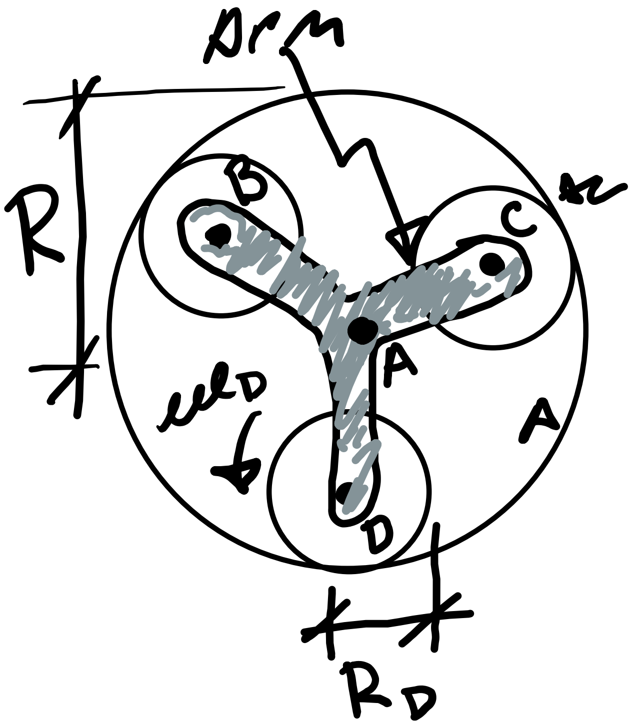
Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$



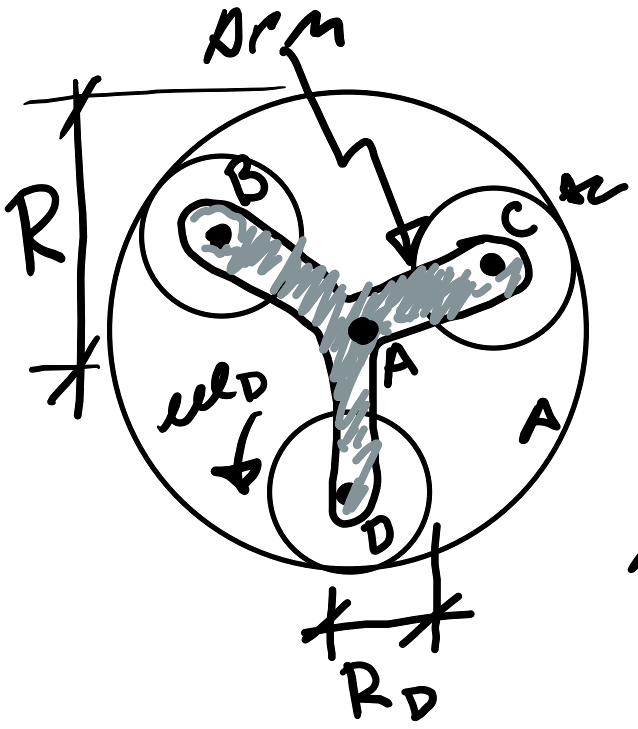
Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$



Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

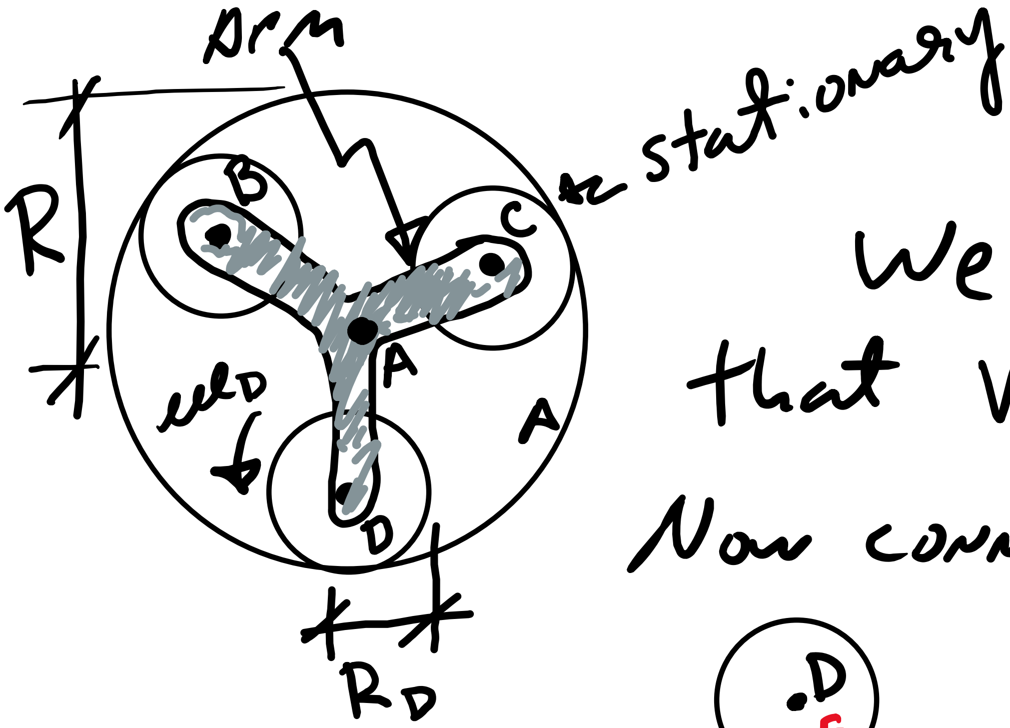


Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots:

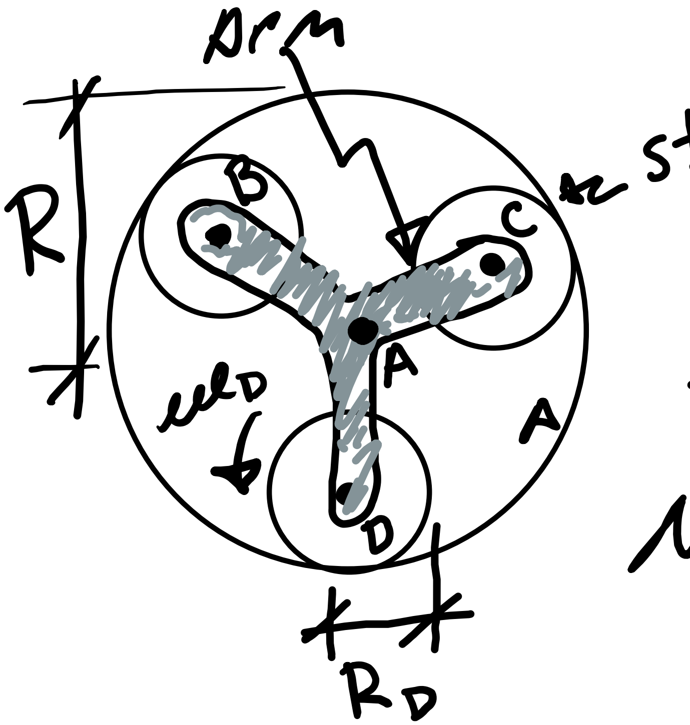
Example: Find ω_{ARM}



We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots

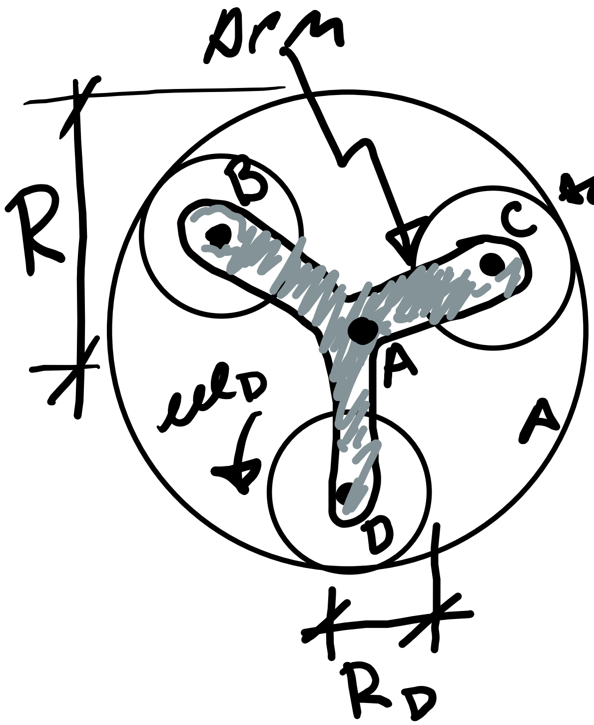
Example: Find ω_{ARM}



We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots

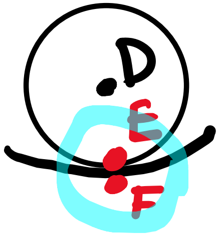
$$\vec{V}_D = \vec{V}_{D/E} + \vec{V}_E$$



Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

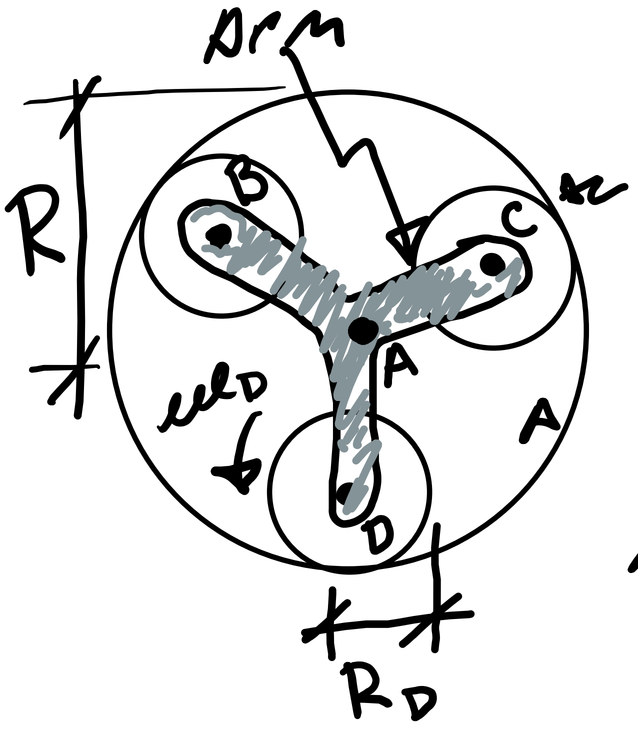
Now connect the dots



$$\vec{V}_D = \vec{V}_{D/E} + \vec{V}_E$$

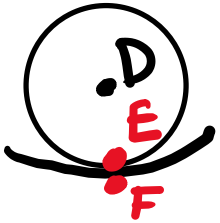
$$\& \ \vec{V}_E = \vec{V}_{E/F} + \vec{V}_F$$

Example: Find ω_{ARM}

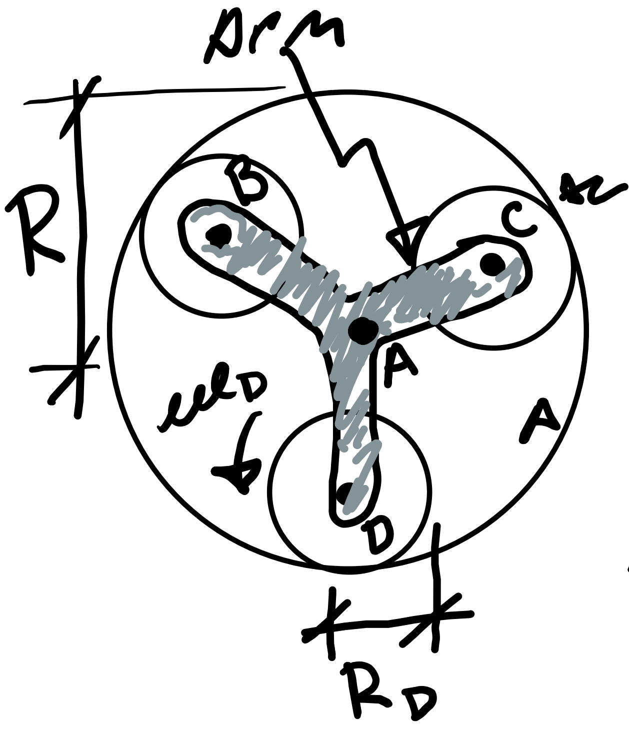


We want $V_{D/A}$ & know that $V_A = \cancel{0}$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots



$$\left.\begin{aligned} \vec{V}_D &= \vec{V}_{D/E} + \vec{V}_E \\ \& \ \vec{V}_E &= \vec{V}_{E/F} + \vec{V}_F \end{aligned}\right\} \text{But } V_F = \cancel{0}$$



Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots

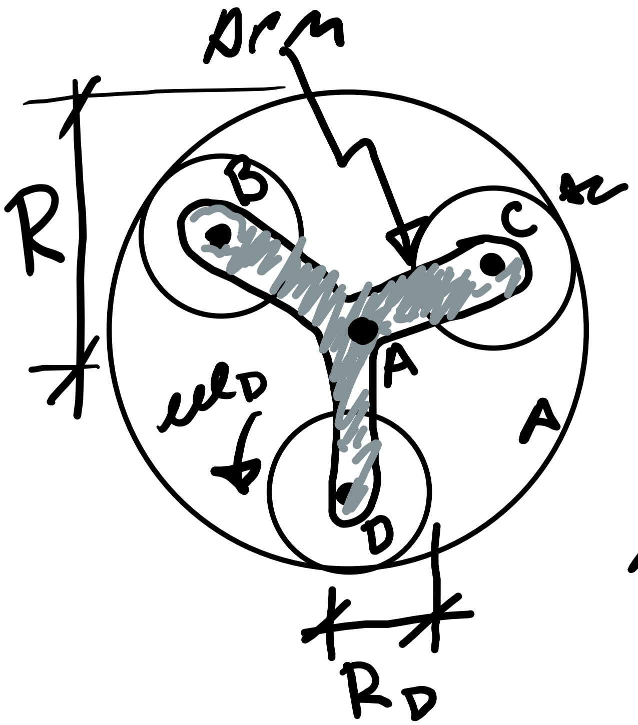
$$\left.\begin{aligned} \vec{V}_D &= \vec{V}_{D/E} + \vec{V}_E \\ \& \ \vec{V}_E &= \vec{V}_{E/F} + \vec{V}_F \end{aligned}\right\}$$

But $V_F = 0$

& $V_{E/F} = 0$

Roll no slip

Example: Find ω_{ARM}



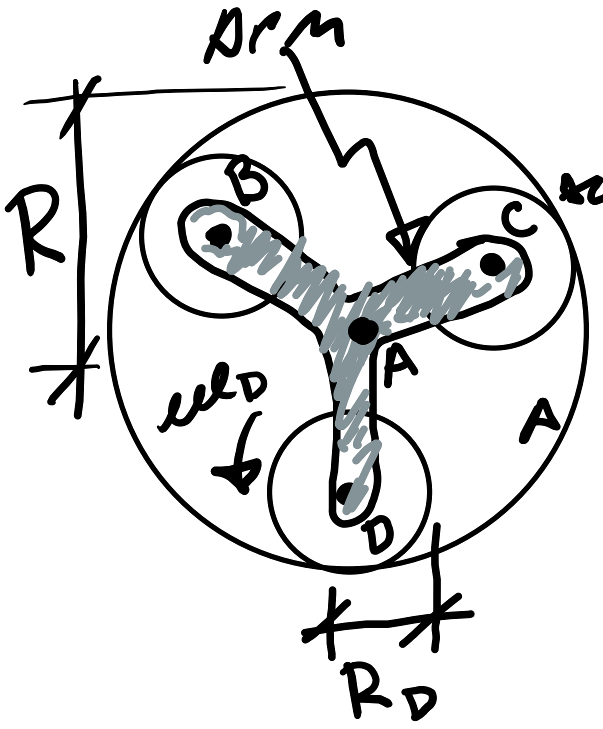
We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots

$$\vec{V}_D = \vec{V}_{D/E} + \vec{V}_E$$

$$\& \; \overset{0}{\cancel{\vec{V}_E}} = \vec{V}_{E/F} + \vec{V}_F$$

But $V_F = 0$ & $V_{E/F} = 0$



stationary

Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec V_D = \vec V_{D/A}$ (1)

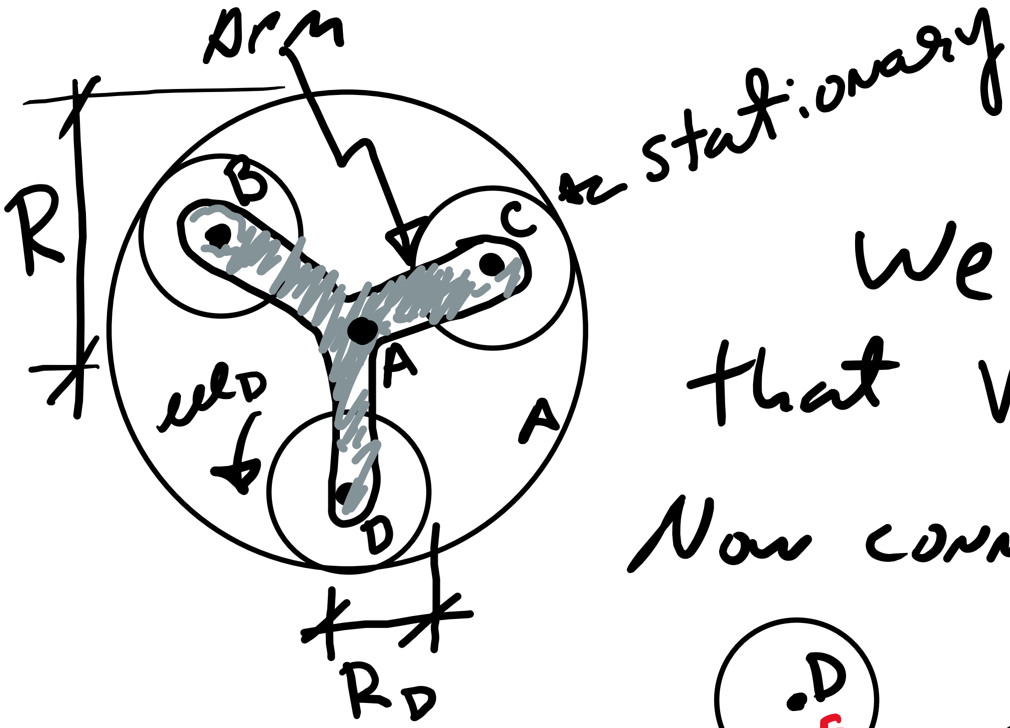
Now connect the dots

$$\vec V_D = \vec V_{D/E} + \cancelto{0}{\vec V_E}$$

$$\& \ \cancelto{0}{\vec V_E} = \vec V_{E/F} + \vec V_F$$

But $V_F = 0$ & $V_{E/F} = 0$

Example: Find ω_{ARM}



We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

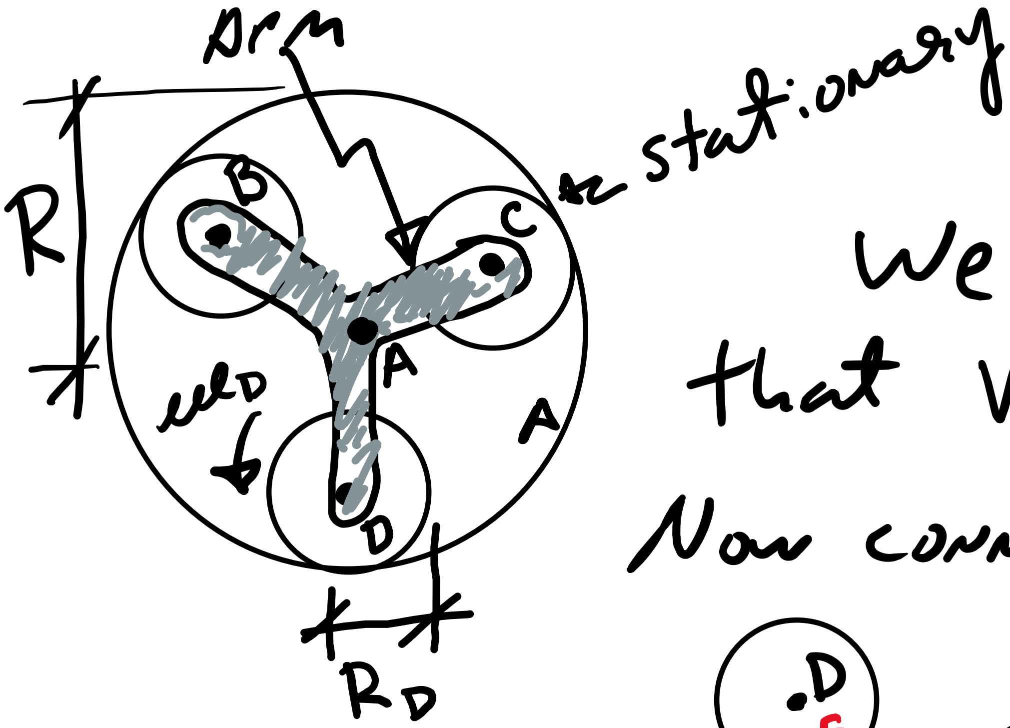
Now connect the dots

$$\vec{V}_D = \vec{V}_{D/E} + \overset{0}{\cancel{\vec{V}_E}}$$

$$\& \ \overset{0}{\cancel{\vec{V}_E}} = \vec{V}_{E/F} + \vec{V}_F$$

But $V_F = 0$ & $V_{E/F} = 0$

so $\vec{V}_D = \vec{V}_{D/E}$



Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

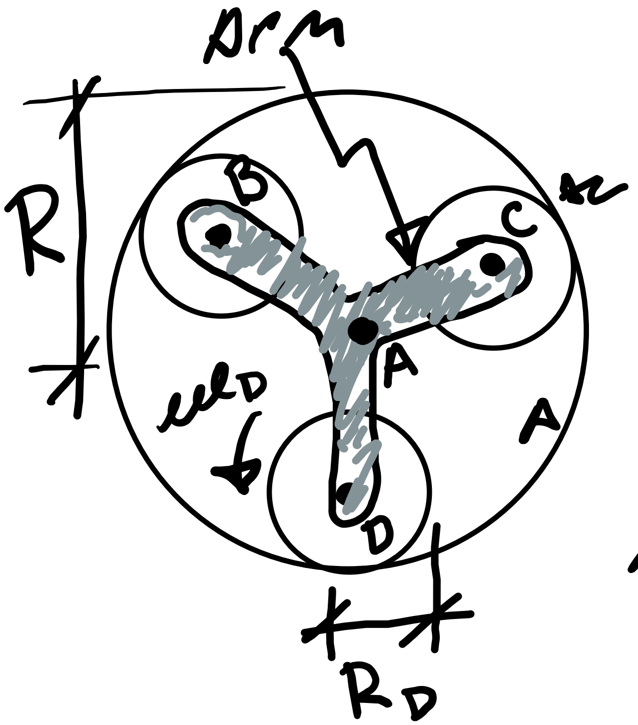
Now connect the dots

$$\vec{V}_D = \vec{V}_{D/E} + \overset{0}{\cancel{\vec{V}_E}}$$

$$\& \ \overset{0}{\cancel{\vec{V}_E}} = \vec{V}_{E/F} + \vec{V}_F$$

But $V_F = 0$ & $V_{E/F} = 0$

So $\vec{V}_D = \vec{V}_{D/E}$ (2)



Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots

$$\vec{V}_D = \vec{V}_{D/E} + \overset{0}{\cancel{\vec{V}_E}}$$

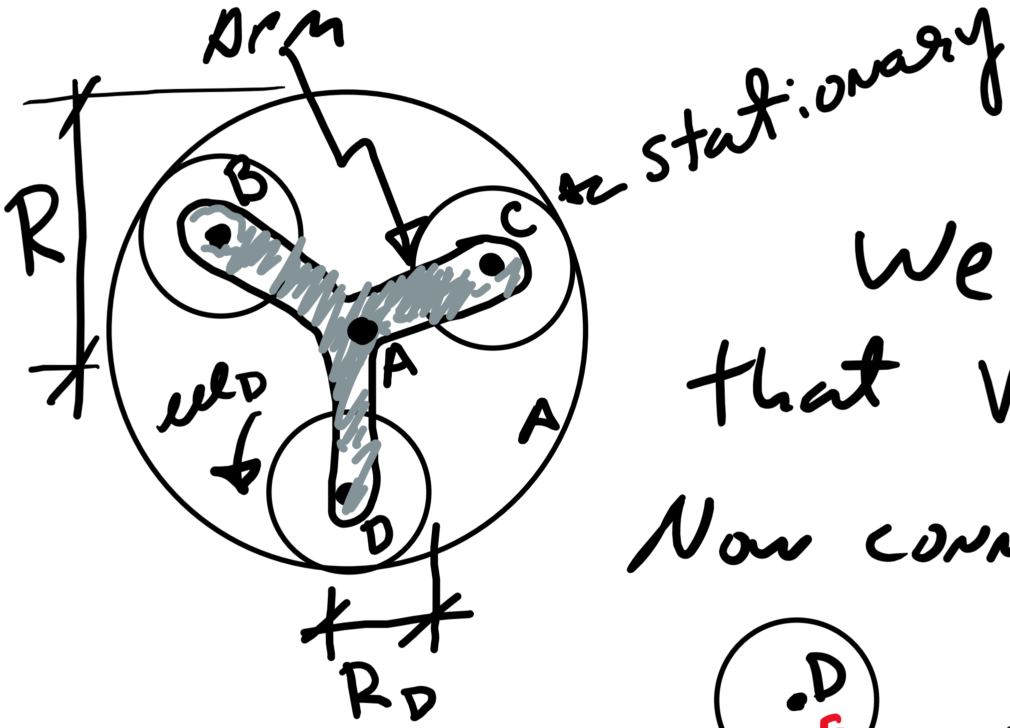
$$\& \ \overset{0}{\cancel{\vec{V}_E}} = \vec{V}_{E/F} + \vec{V}_F$$

But $V_F = 0$ & $V_{E/F} = 0$

so $\vec{V}_D = \vec{V}_{D/E}$ (2)

Equs 1&2 give us

$$\vec{V}_{D/E} = \vec{V}_{D/A}$$



Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots

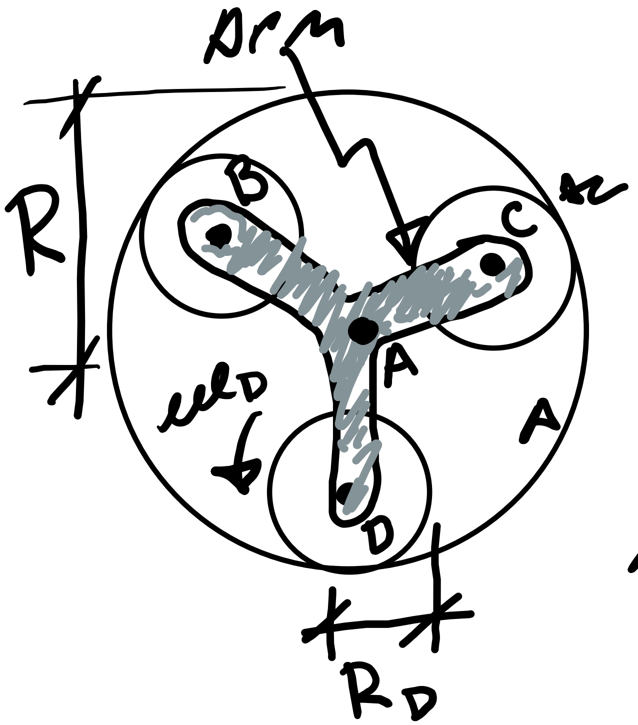
$\vec{V}_D = \vec{V}_{D/E} + \overset{0}{\cancel{\vec{V}_E}}$

& $\overset{0}{\cancel{\vec{V}_E}} = \vec{V}_{E/F} + \vec{V}_F$ But $V_F = 0$ & $V_{E/F} = 0$

So $\vec{V}_D = \vec{V}_{D/E}$ (2) Eqns 1&2 give us

$\vec{V}_{D/E} = \vec{V}_{D/A}$ But $V_{D/E} = R_D \omega_D$

Example: Find ω_{ARM}



We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots

$$\vec{V}_D = \vec{V}_{D/E} + \cancel{\vec{V}_E}^{\,0}$$

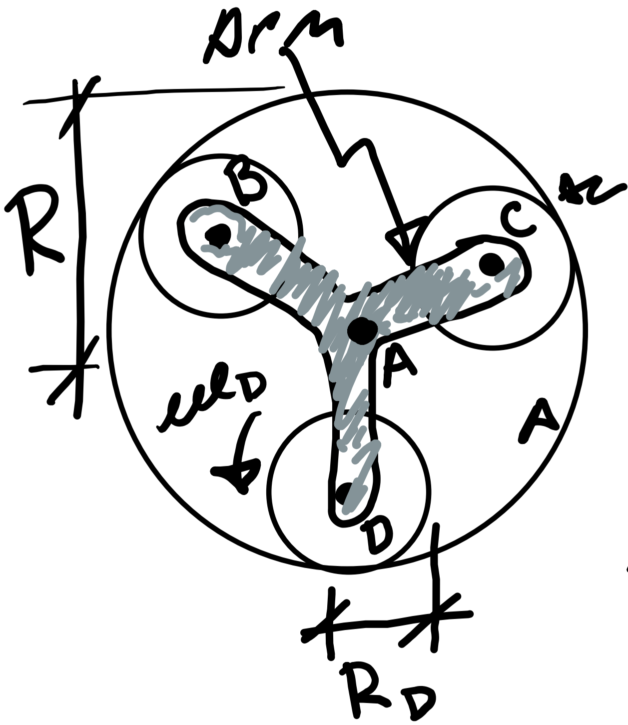
$$\& \ \cancel{\vec{V}_E}^{\,0} = \vec{V}_{E/F} + \vec{V}_F$$

But $V_F = 0$ & $V_{E/F} = 0$

So $\vec{V}_D = \vec{V}_{D/E}$ (2)

Eqns 1&2 give us

$\vec{V}_{D/E} = \vec{V}_{D/A}$ But $V_{D/E} = R_D\,\omega_D$ & $V_{D/A} = r_{D/A}\,\omega_{ARM}$



Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots

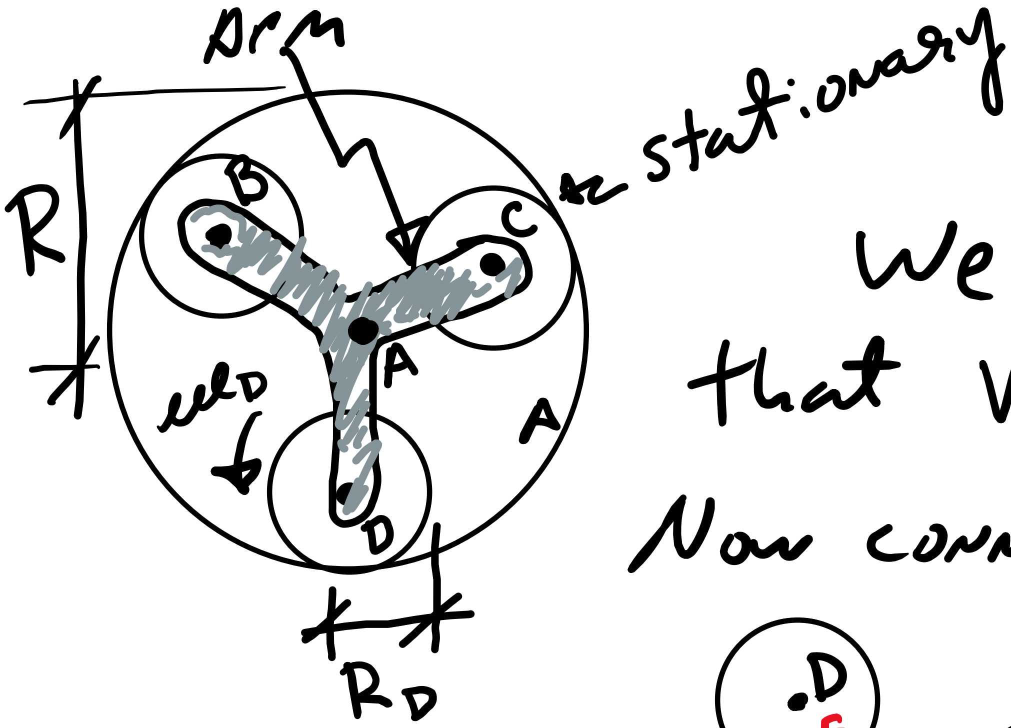
$$\left.\begin{array}{l}\vec{V}_D = \vec{V}_{D/E} + \overset{0}{\cancel{\vec{V}_E}} \\ \& \ \overset{0}{\cancel{\vec{V}_E}} = \vec{V}_{E/F} + \vec{V}_F\end{array}\right\}$$

But $V_F = 0$ & $V_{E/F} = 0$

So $\vec{V}_D = \vec{V}_{D/E}$ (2)   Eqns 1&2 give us

$\vec{V}_{D/E} = \vec{V}_{D/A}$ But $V_{D/E} = R_D \omega_D$ & $V_{D/A} = r_{D/A} \omega_{ARM}$

So $\omega_{ARM}(R-R_D) = \omega_D R_D$



Example: Find ω_{ARM}

We want $V_{D/A}$ & know that $V_A = 0$ so $\vec{V}_D = \vec{V}_{D/A}$ (1)

Now connect the dots

$$\left.\begin{aligned}\vec{V}_D &= \vec{V}_{D/E} + \overset{0}{\cancel{\vec{V}_E}} \\ \& \; \overset{0}{\cancel{\vec{V}_E}} &= \vec{V}_{E/F} + \vec{V}_F\end{aligned}\right\}$$

But $V_F = 0$ & $V_{E/F} = 0$

So $\vec{V}_D = \vec{V}_{D/E}$ (2) Eqns 1&2 give us

$\vec{V}_{D/E} = \vec{V}_{D/A}$ But $V_{D/E} = R_D\omega_D$ & $V_{D/A} = r_{D/A}\omega_{ARM}$

So $\omega_{ARM}(R-R_D) = \omega_D R_D \Rightarrow \boxed{\omega_{ARM} = \dfrac{R_D\omega_D}{(R-R_D)}}$

The method of connecting the dots should always work, The method of connecting the dots should always work, however, if you feel confident in your understanding, The method of connecting the dots should always work, however, if you feel confident in your understanding, you could perform the previous problem quickly.

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The more complicated the problem

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The more complicated the problem, the more likely you will need to connect the dots methodically

Kinetics :

Kinetics : $\Sigma \vec{F} = m\vec{a}$ & $\Sigma \vec{M}_G = \dot{\vec{H}}_G$,

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Kinetics: $\Sigma\vec{F}=m\vec{a}$ & $\Sigma\vec{M}_G=\dot{\vec{H}}_G$,

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For rigid body:

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For rigid body: $\vec{H}_G = \bar{I}\vec{\omega} \Rightarrow \dot{\vec{H}}_G = \bar{I}\vec{\alpha}$

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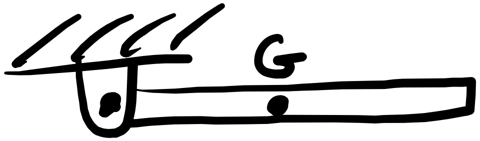
$\Rightarrow \sum \vec{M}_G = \bar{I}\vec{\alpha}$ For rotation about fixed point C: $\sum \vec{M}_C = I_C\vec{\alpha}$,

Kinetics

: $\Sigma\vec{F} = m\vec{a}$ & $\Sigma\vec{M}_G = \dot{\vec{H}}_G$, where $\vec{M} = \vec{r}\times\vec{F}$ & $\vec{H} = \vec{r}\times\vec{L}$ & $\vec{L} = m\vec{v}$

For rigid body: $\vec{H}_G = \bar{I}\vec{\omega} \Rightarrow \dot{\vec{H}}_G = \bar{I}\vec{\alpha}$ $\Rightarrow \Sigma\vec{M}_G = \bar{I}\vec{\alpha}$ For rotation about fixed point C: $\Sigma\vec{M}_c = I_c\vec{\alpha}$, where $I_c = \bar{I} + m r^2_{G/c}$

Example problem:



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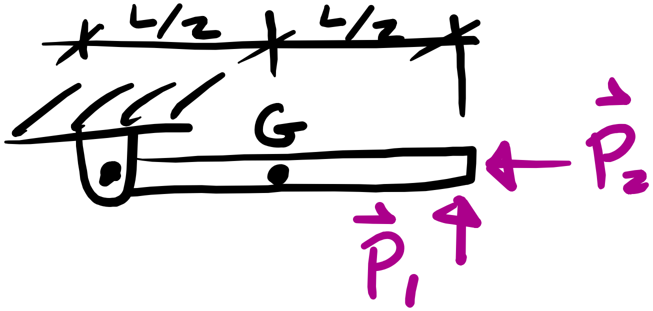

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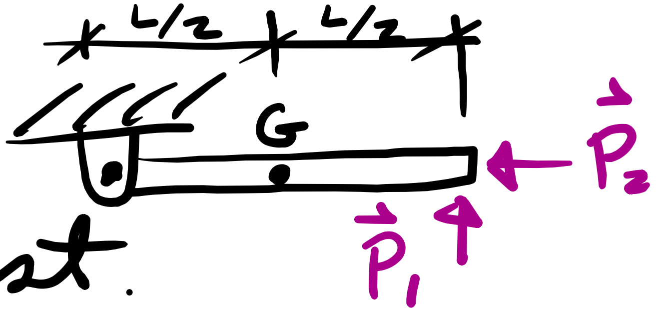
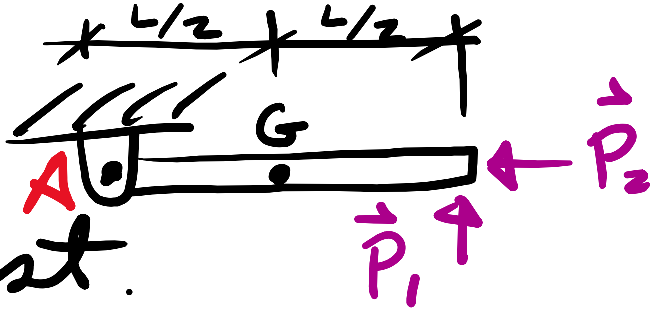


Arm is initially at rest.

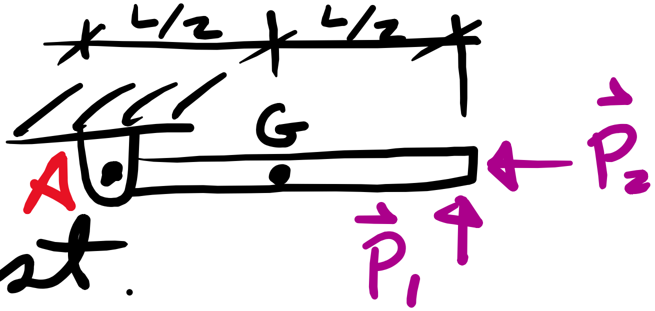
Example problem:



Arm is initially at rest.

Find reaction force at A

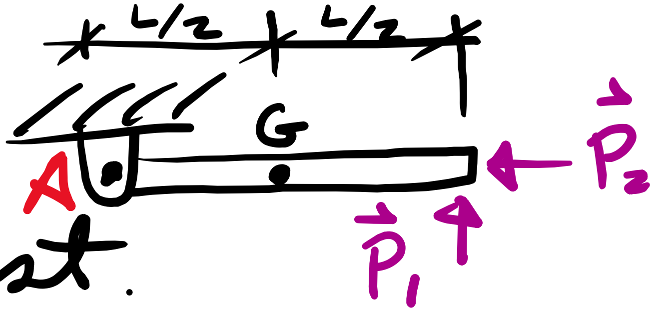
Example problem:



Arm is initially at rest.

Find reaction force at A & α_{ARM}

Example problem:



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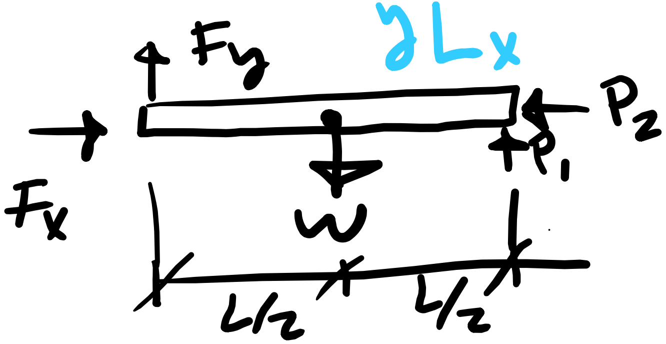

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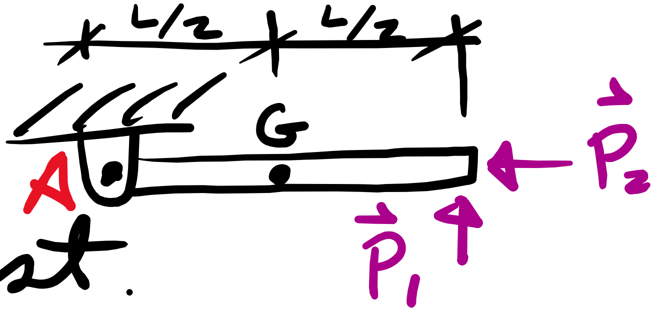

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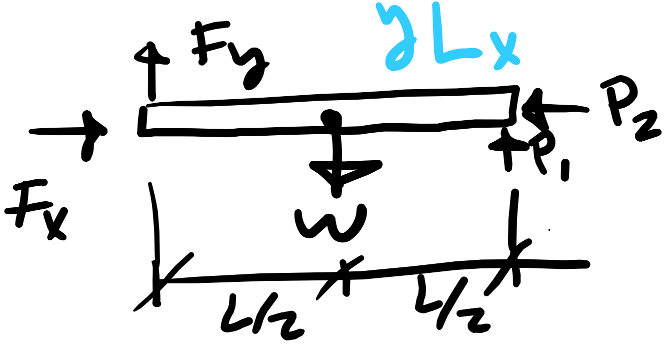
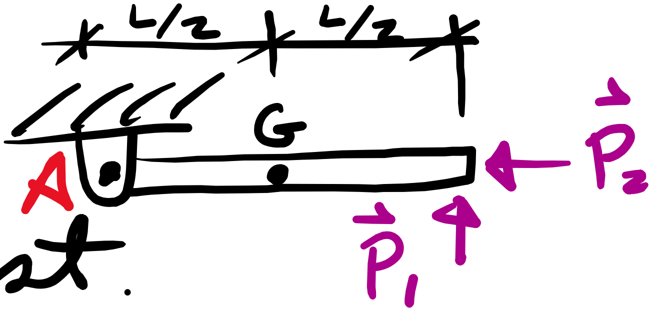


$$\Sigma F_x = m\bar{a}_x$$

Example problem:



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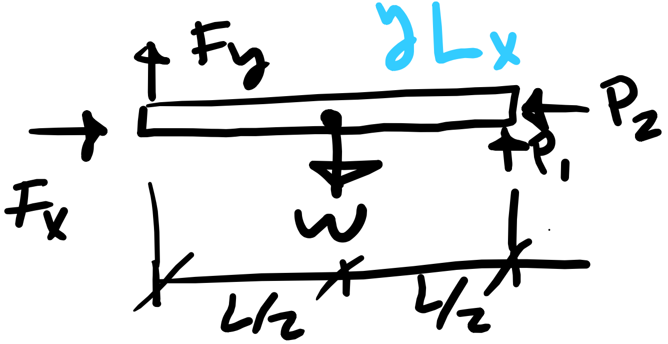

$\Sigma F_x = m\cancel{a_x}$

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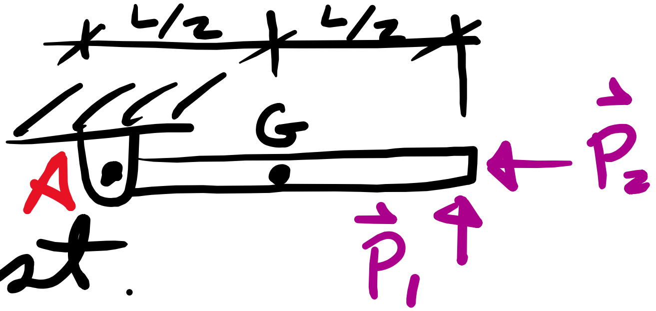

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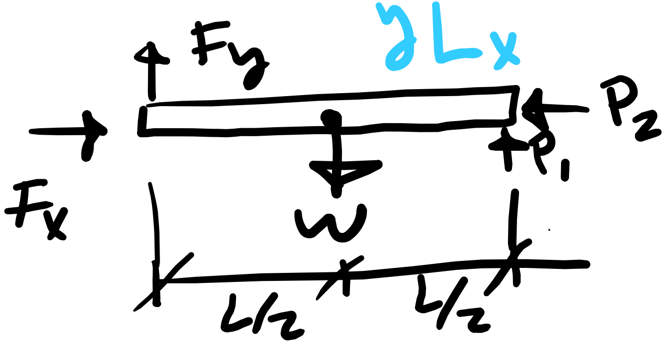
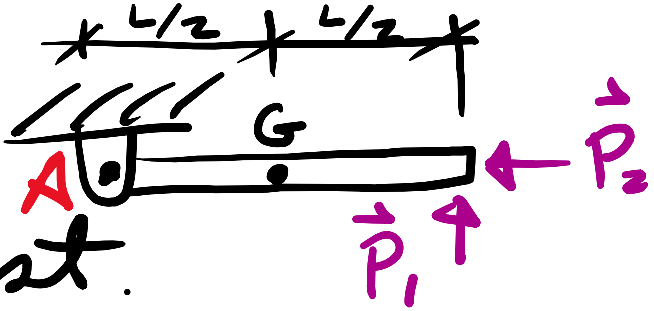


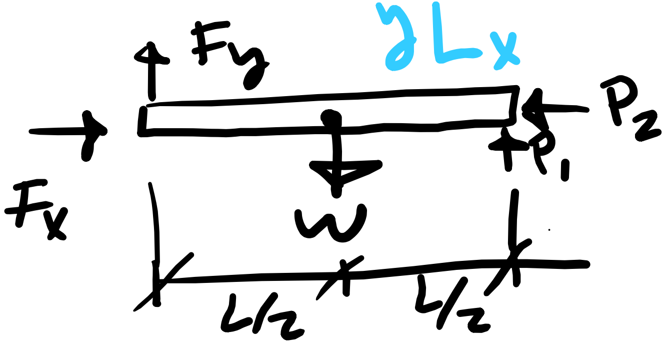
$$\Sigma F_x = m\cancel{a_x} \Rightarrow \boxed{F_x = P_2}$$

Example problem:



Arm is initially at rest.

Find reaction force at A & α_{arm}



$$\sum F_x = m\cancel{a_x} \Rightarrow \boxed{F_x = P_2} \quad (1)$$

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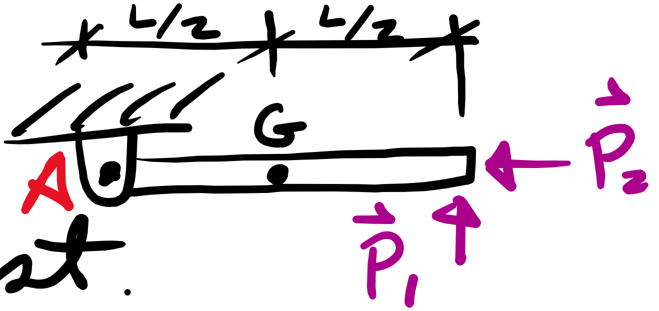

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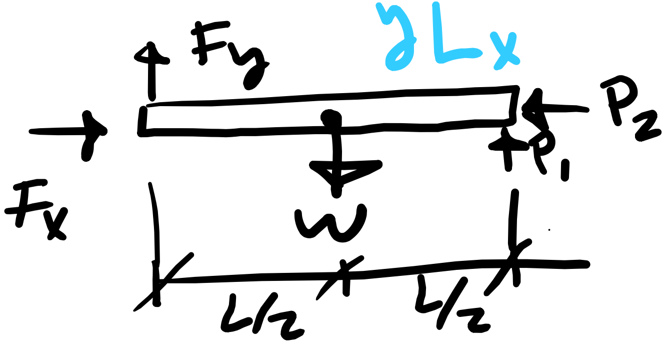
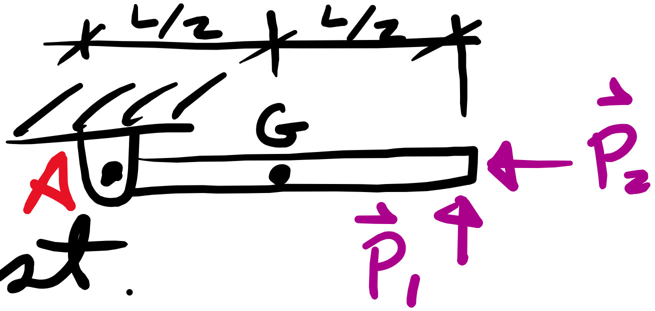


$$\Sigma F_x = m\cancelto{0}{\bar{a}_x} \Rightarrow \boxed{F_x = P_2} \quad (1)$$

$$\Sigma F_y = m\bar{a}_y$$

Example problem:



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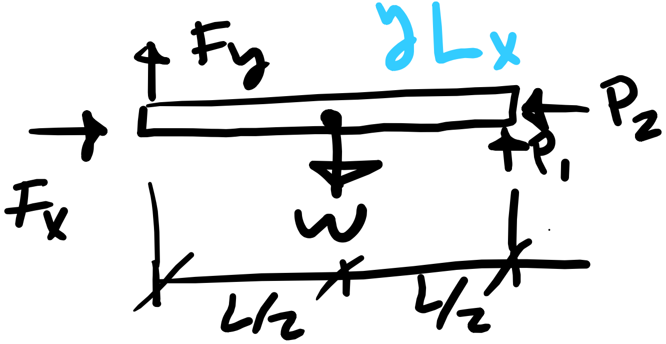

$$\Sigma F_x = m\cancel{\bar{a}_x} \Rightarrow \boxed{F_x = P_2} \quad (1)$$

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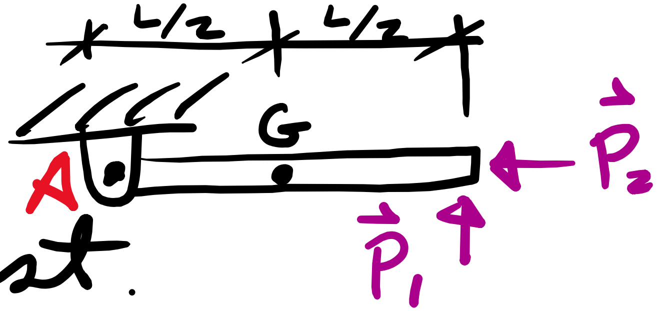

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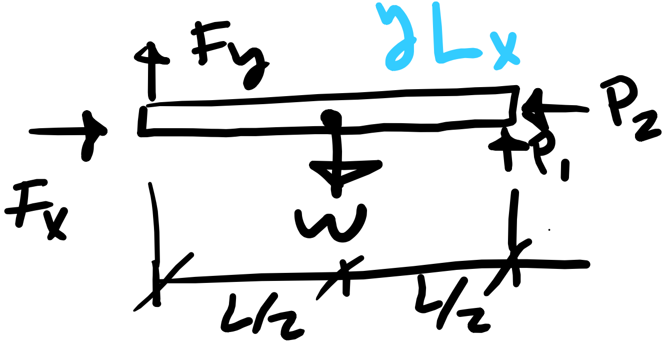
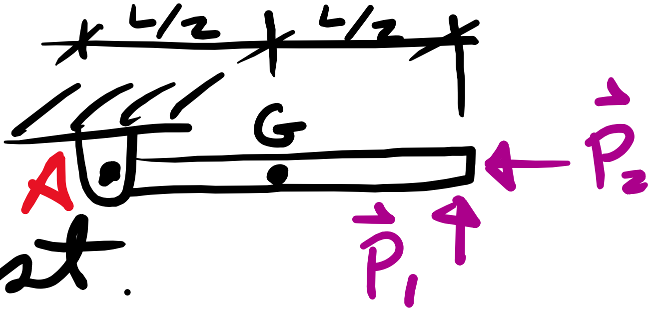


$\Sigma F_x = m\cancelto{0}{\bar{a}_x} \Rightarrow \boxed{F_x = P_2}$ (1)

$\Sigma F_y = m\bar{a}_y \Rightarrow F_y + P_1 - W = m\bar{a}_y$

But $\bar{a}_y = \alpha L/2$

Example problem:



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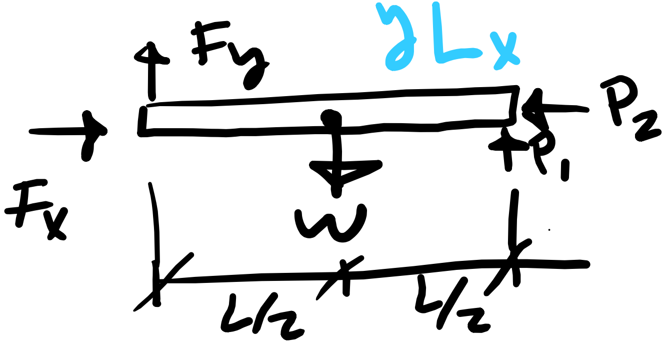


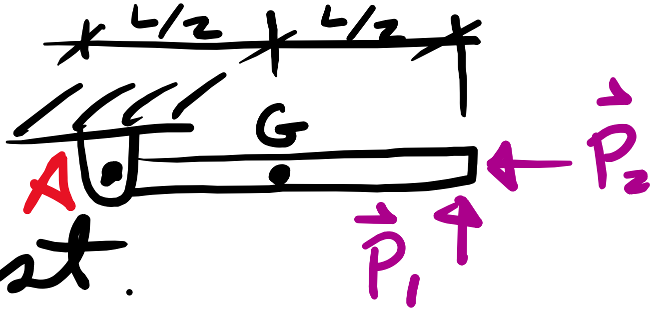
$\Sigma F_x = m\cancelto{0}{\bar{a}_x} \Rightarrow \boxed{F_x = P_2}$ (1)

$\Sigma F_y = m\bar{a}_y \Rightarrow F_y + P_1 - W = m\bar{a}_y$

But $\bar{a}_y = \alpha \frac{L}{2} \neq 0$

$F_y + P_1 - W = m\left(\frac{L}{2}\right)\alpha$

Example problem:



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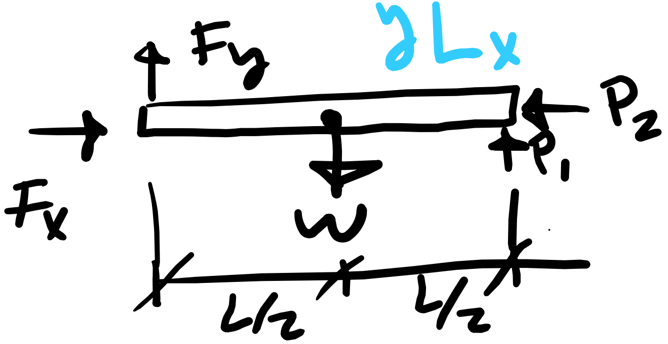

$\Sigma F_x = \cancel{m\bar{a}_x} \Rightarrow \boxed{F_x = P_2}$ (1)

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But $\bar{a}_y = \alpha L/2$ so

$F_y + P_1 - W = m\left(\frac{L}{2}\right)\alpha$ (2)

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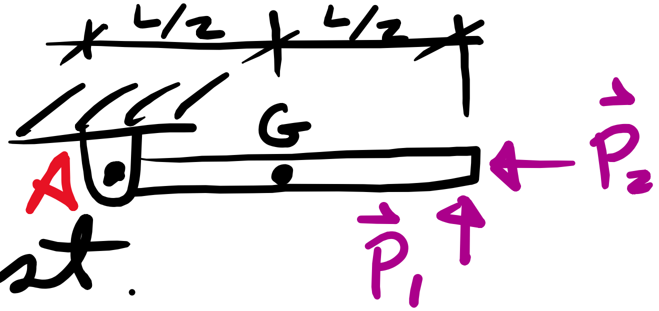

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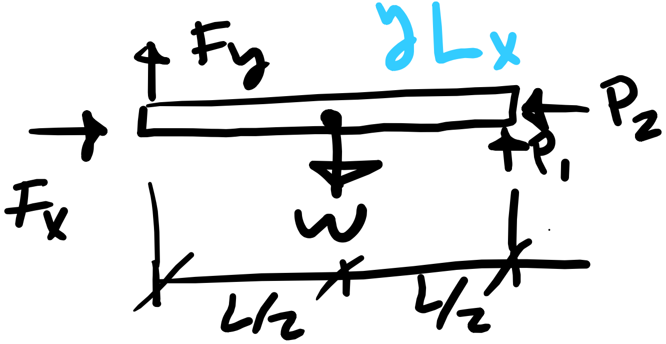

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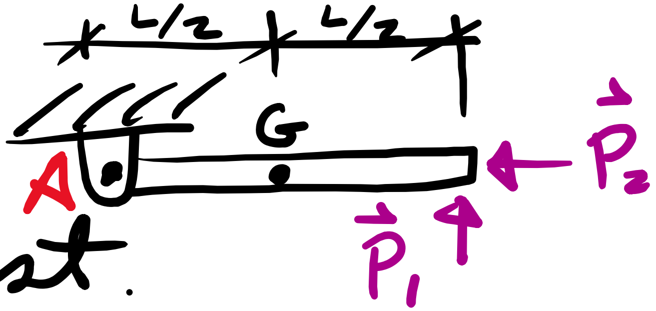
$\circlearrowleft \Sigma M_A = I_A \alpha$

Example problem:



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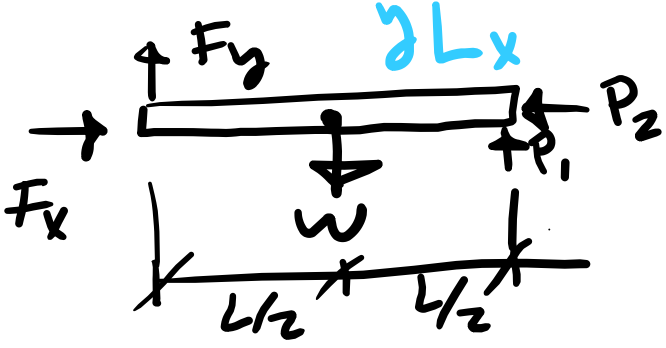

$\Sigma F_x = m\cancel{\bar{a}_x} \Rightarrow \boxed{F_x = P_2}$ (1)

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But $\bar{a}_y = \alpha \frac{L}{2} \neq 0$

$F_y + P_1 - W = m\left(\frac{L}{2}\right)\alpha$ (2) $\circlearrowleft \Sigma M_A = I_A \alpha \Rightarrow$

$-W\frac{L}{2} + P_1 L = I_A \alpha$

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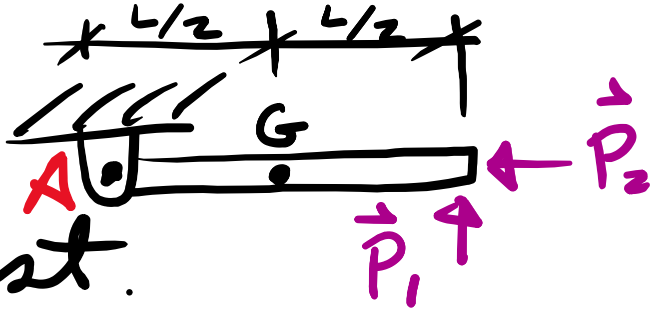

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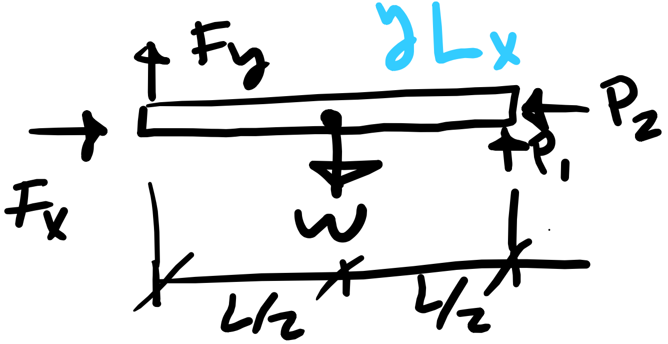

$\Sigma F_x = m\bar{a}_x \Rightarrow \boxed{F_x = P_2}$ (1)

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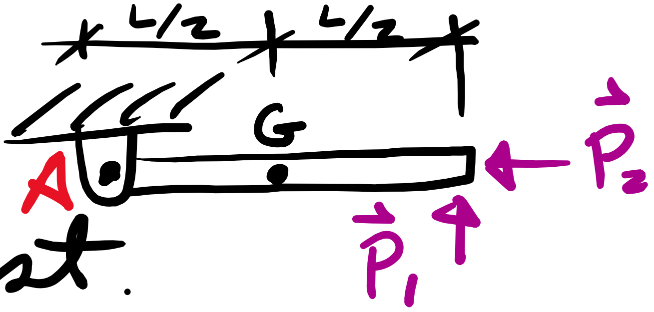
But $\bar{a}_y = \alpha \frac{L}{2}$ so

$F_y + P_1 - W = m\left(\frac{L}{2}\right)\alpha$ (2) $\quad \circlearrowleft \Sigma M_A = I_A\alpha \Rightarrow$

$-W\frac{L}{2} + P_1 L = I_A\alpha \Rightarrow \alpha = \frac{P_1 L - W\frac{L}{2}}{I_A}$

Example problem:



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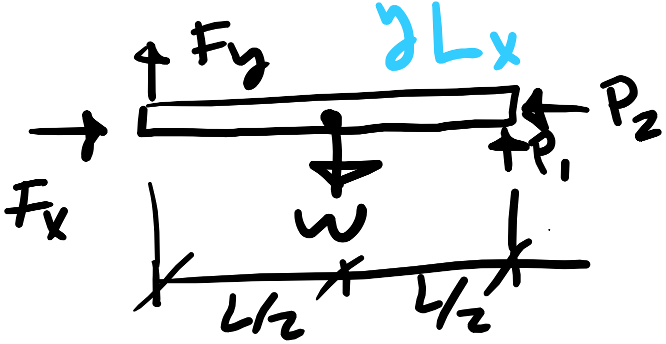

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$I_A = \frac{mL^2}{12} + m\frac{L^2}{4}$

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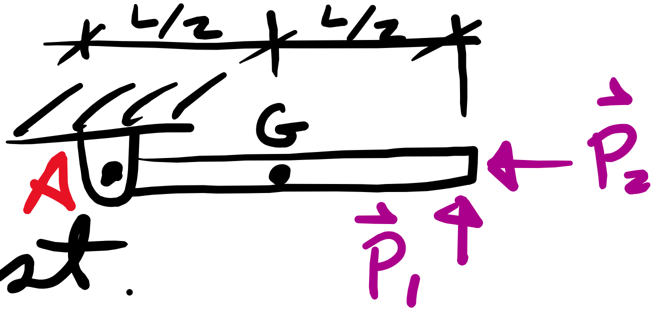

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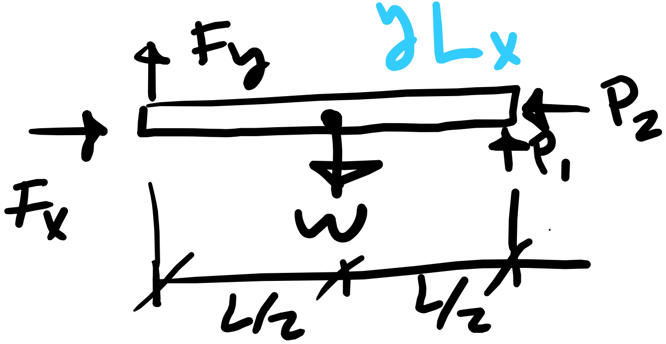

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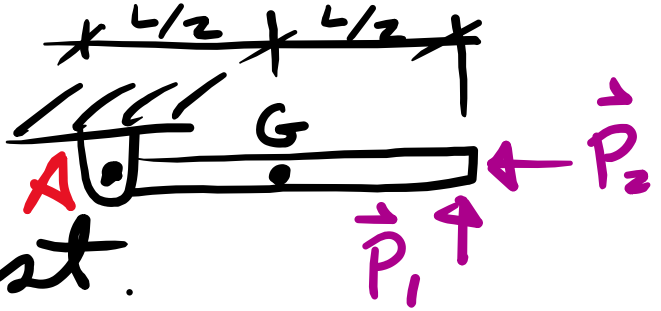
But $\bar{a}_y = \alpha \frac{L}{2} \neq 0$

$$F_y + P_1 - W = m\left(\frac{L}{2}\right)\alpha \quad (2) \qquad \circlearrowleft \Sigma M_A = I_A \alpha \Rightarrow$$

$$-W\frac{L}{2} + P_1 L = I_A \alpha \Rightarrow \alpha = \frac{P_1 L - W\frac{L}{2}}{I_A}$$ But

$$I_A = \frac{mL^2}{12} + m\frac{L^2}{4} = mL^2/3$$

Example problem:



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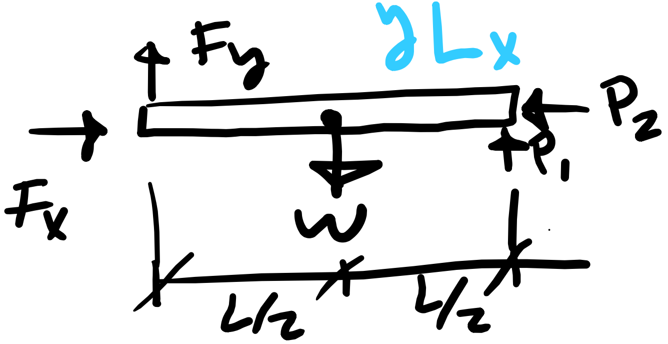
Find reaction force at A & α_{arm}



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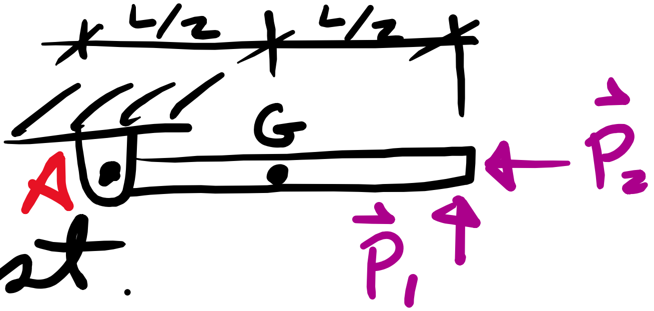
so

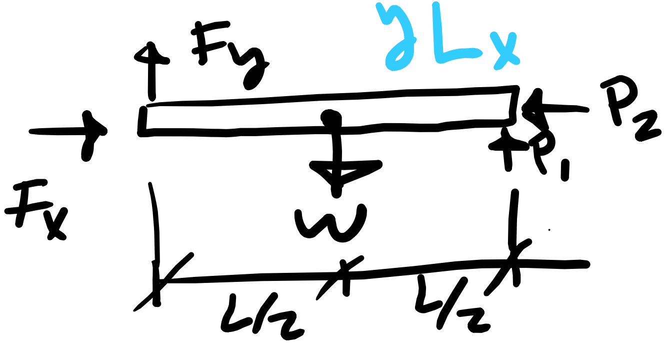
$$\boxed{\alpha = \left[\frac{P_1 - W/2}{ML/3}\right] \circlearrowleft}$$

Example problem:



Arm is initially at rest.

Find reaction force at A & α_{ARM}



$\Sigma F_x = m\cancelto{0}{\bar{a}_x} \Rightarrow \boxed{F_x = P_2}$ (1)

$\Sigma F_y = m\bar{a}_y \Rightarrow F_y + P_1 - W = m\bar{a}_y$

But $\bar{a}_y = \alpha \frac{L}{2}$ so

$F_y + P_1 - W = m\left(\frac{L}{2}\right)\alpha$ (2) $\circlearrowleft \Sigma M_A = I_A\alpha \Rightarrow$

$-W\frac{L}{2} + P_1 L = I_A\alpha \Rightarrow \alpha = \frac{P_1 L - W\frac{L}{2}}{I_A}$ But

$I_A = \frac{mL^2}{12} + m\frac{L^2}{4} = mL^2/3$ so

$$\boxed{\alpha = \left[\frac{P_1 - W/2}{mL/3}\right]} \; 3$$

Equation 2 now gives $F_y = W - P + \left(\frac{3}{2}\right)\left[P_1 - W/2\right]$

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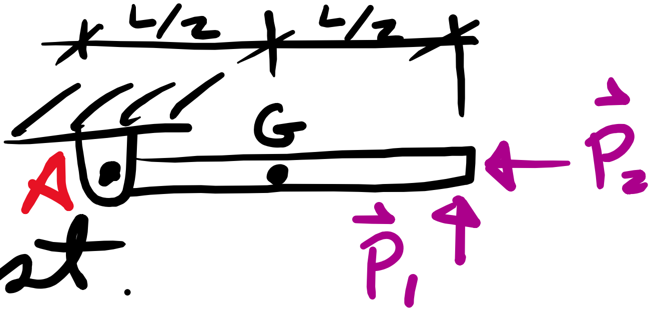

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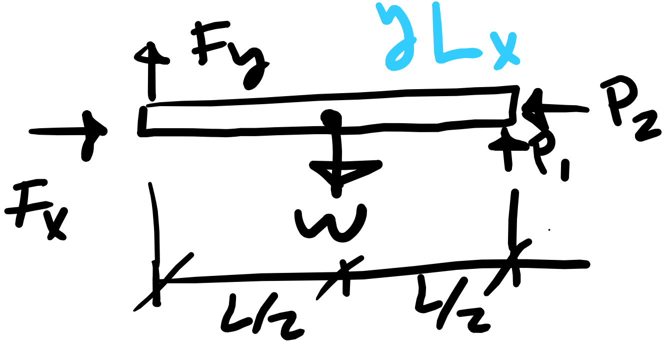

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$\Rightarrow \boxed{F_y = \frac{W}{4} + \frac{P_1}{2}}$

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$U_{1\to 2} = \Delta T,$

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$U_{1\to 2} = \Delta T$, for moment couple

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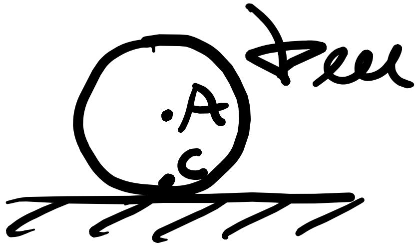

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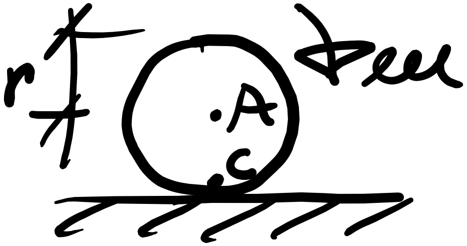

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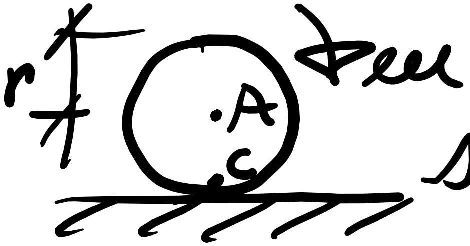

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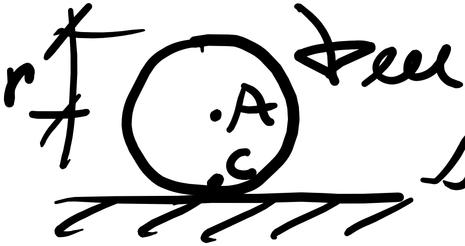

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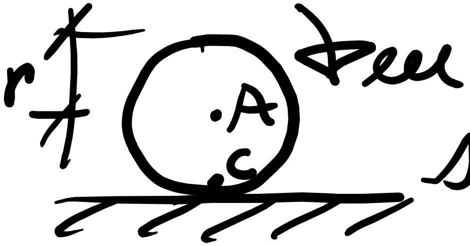

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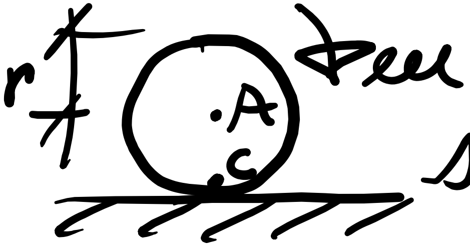

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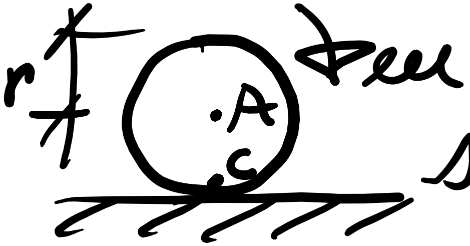

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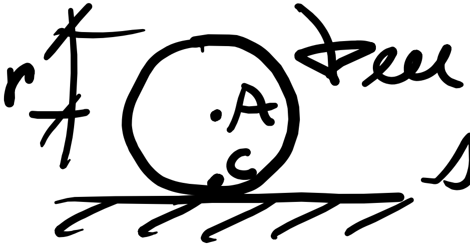

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[Same as before]

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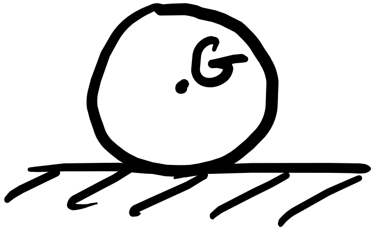

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$$T_I - M\frac{d}{r} = 0 \Rightarrow T_I = M\frac{d}{r} \Rightarrow \boxed{d = \frac{Tr}{M}}$$

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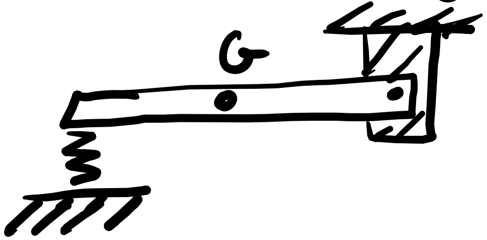

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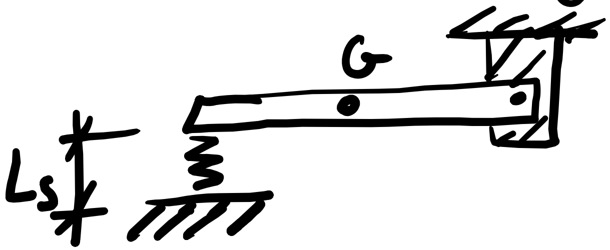

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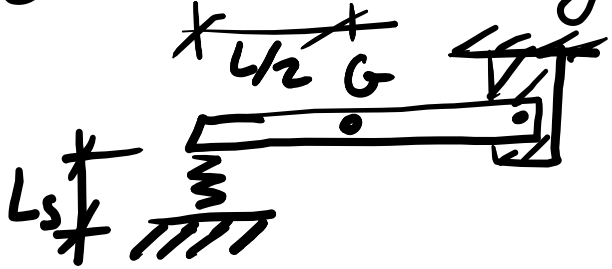

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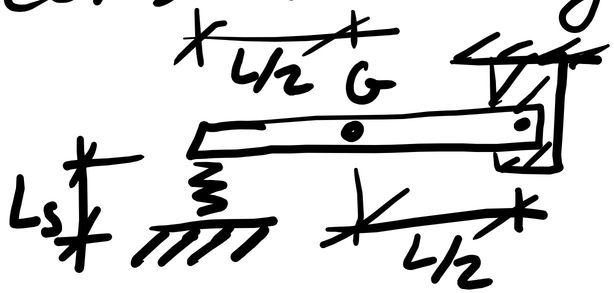

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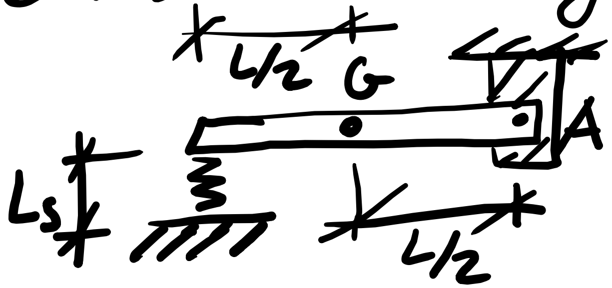

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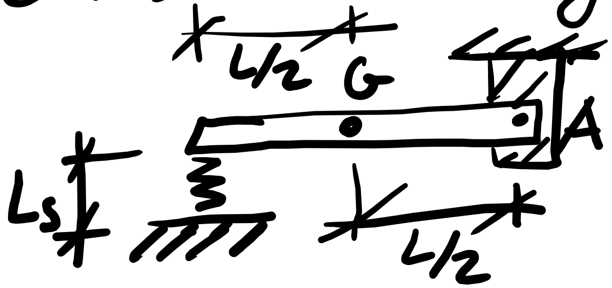

Mass of arm $= m$

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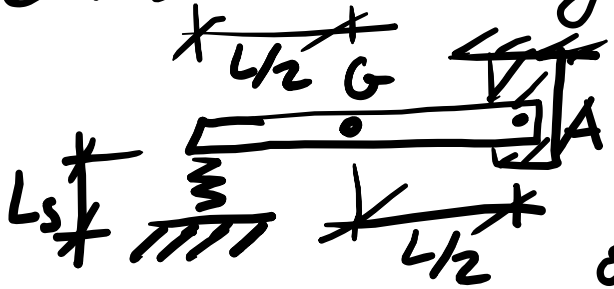

Mass of arm $= m$

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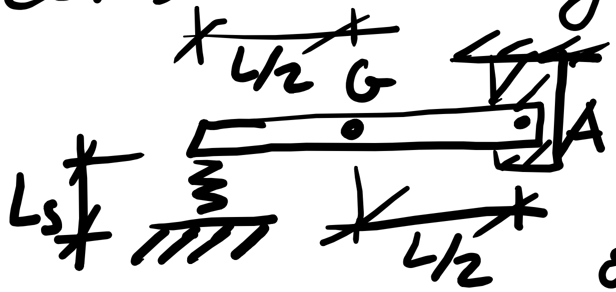

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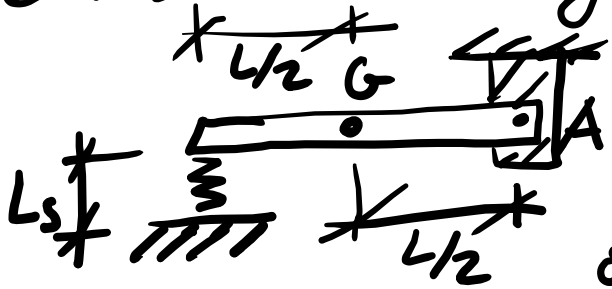

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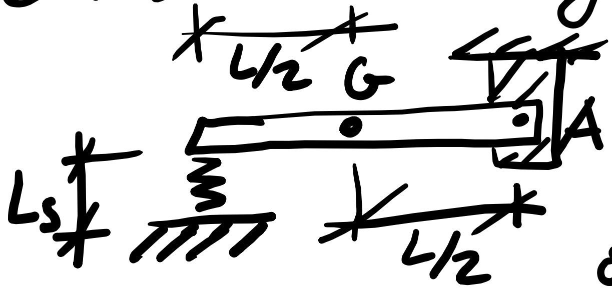

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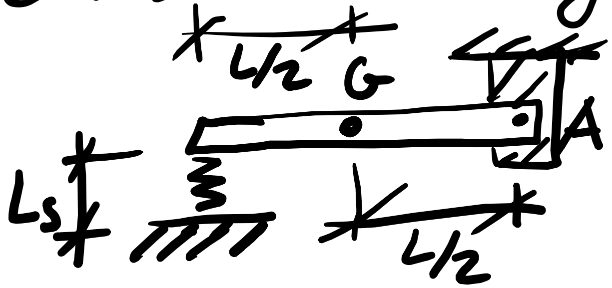

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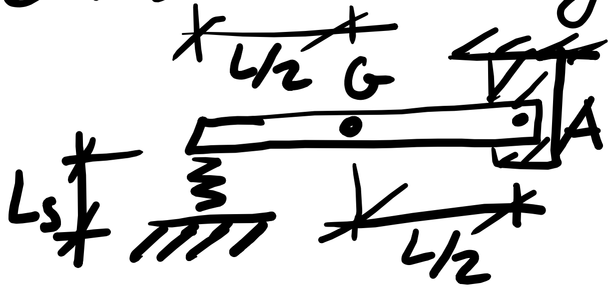

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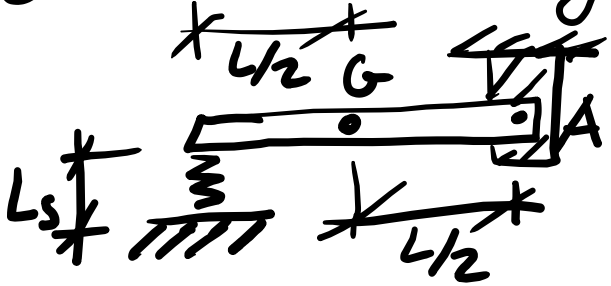

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Energy conservation

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If only conservative forces then $U^{NC}_{I\to F} = 0$

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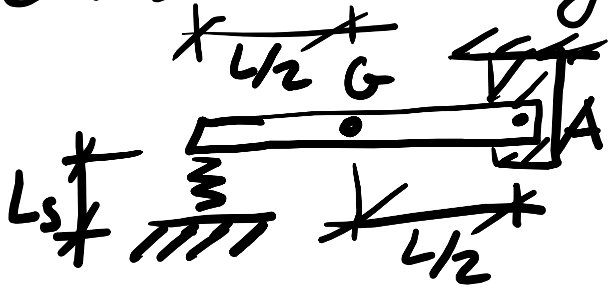

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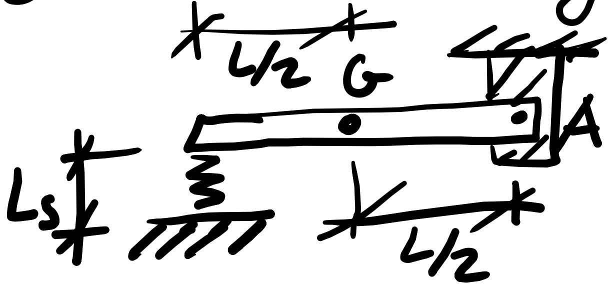

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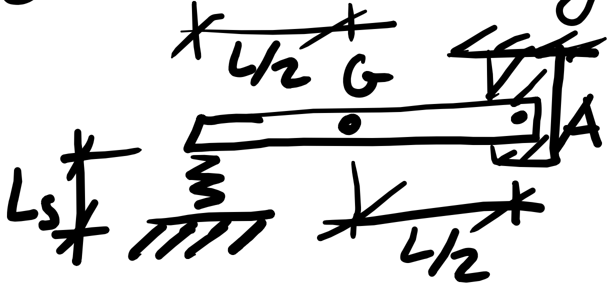

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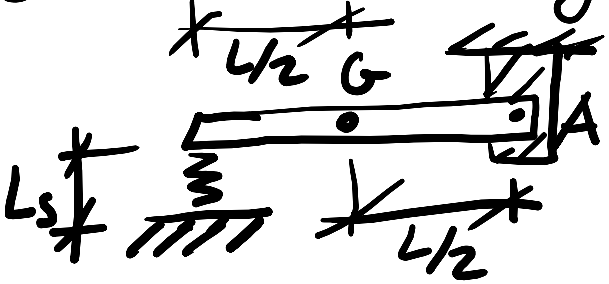

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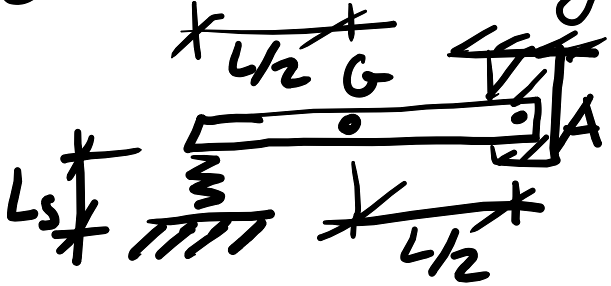

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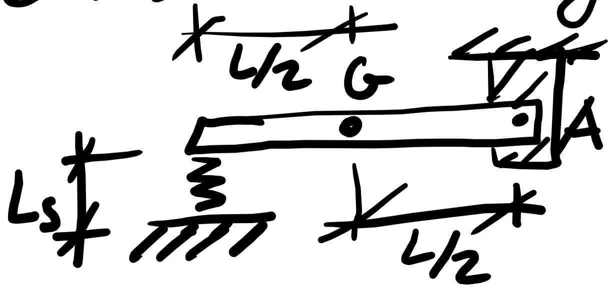

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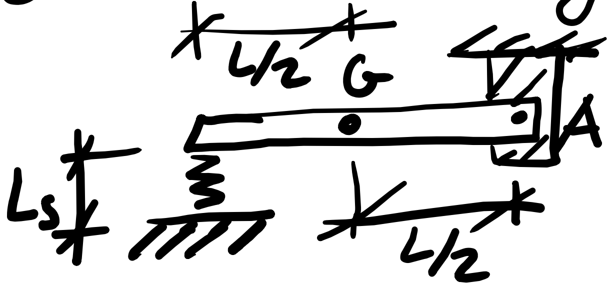

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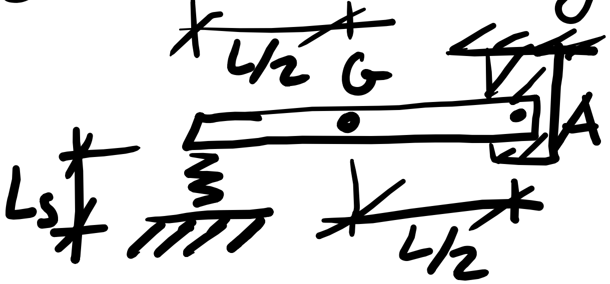

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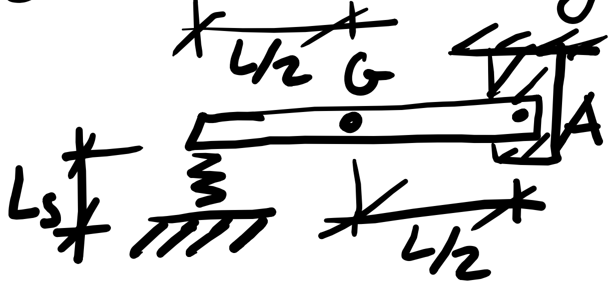

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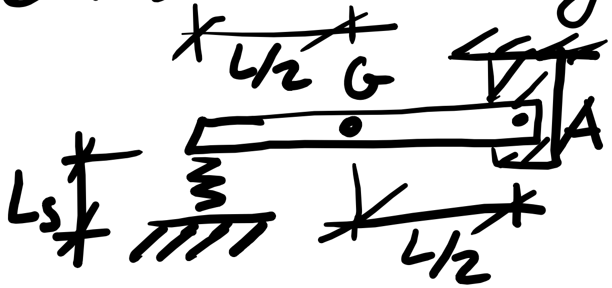

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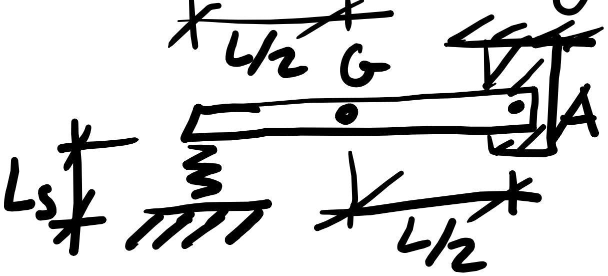

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$$\boxed{\omega_F = \sqrt{\frac{3[k(L_s-L_o)^2 - mgL]}{ML^2}}}$$

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Note: M is a brake so $M < 0$ if $\omega_I > 0$