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EFFECTS OF NEARLY MASSLESS, SPIN ZERO PARTICLES ON LIGHT PROPAGATION IN A MAGNETIC FIELD

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ABSTRACT

Very light or massless spin zero particles coupled to two photons, such as axions or genuine Goldstone bosons, induce small changes in the polarization state of a laser beam travelling in a magnetic field. Severe bounds on mass and coupling can be obtained through the observation of changes in polarization plane and ellipticity of the beam, at the level of accuracy allowed by present technologies. A positive signal would determine mass, coupling and parity of this hypothetical particle.

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Very accurate measurements can be performed on the variation of the polarization state of a light beam. Indeed the attainable precision is such that one may attempt to detect in this way effects related to energy or mass scales so high as to be inaccessible with present accelerators. The experimental situation we shall consider is that[1),2)] of a laser beam propagating in a constant magnetic field of strength B, orthogonal to the beam itself. The beam is linearly polarized at the beginning; after travelling for a length L, which can be made to be of the order of a few kilometres, one can detect very small values of the ellipticity and very small changes in the orientation of the polarization plane.

In this note, we wish to discuss the effects arising from the presence of very light or massless pseudoscalar particles coupled to the electromagnetic field by the effective interaction:

$$\mathcal{L}_P = \frac{1}{4M} \varphi F_{\mu\nu} \tilde{F}^{\mu\nu} \tag{1}$$

with $\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\lambda\sigma}F_{\lambda\sigma}$, the familiar dual electromagnetic tensor. The dimensionful parameter M characterizes the strength of the interaction. Typical examples of such particles are pseudo or real Goldstone bosons [such as the axion[3)] or the arions considered in Ref. 4)], associated to some spontaneously broken chiral symmetry. In this case, M is related to the vacuum expectation value, V, of the field which breaks the symmetry according to:

$$(4M)^{-1} = \text{const.}\ \frac{e\alpha}{V} \tag{2}$$

($\alpha = e^2/4\pi = (137)^{-1}$) the coefficient being of order unity and model dependent. The case of a scalar field, σ, will also be considered for completeness, with the effective interaction:

$$\mathcal{L}_S = \frac{1}{4M_S} \sigma F_{\mu\nu} F^{\mu\nu} \tag{3}$$

As we shall see, with the presently available technologies, and for very small values of the pseudoscalar mass, $m \lesssim 10^{-4}$ eV, one can be sensitive to values of $M(M_S)$ as high as 10^{10} GeV. These values compare favourably with the lower bounds one can set for M from astrophysical considerations[5)-7)] with all the advantages inherent to a laboratory experiment. The detection of a signal in the two effects (rotation of polarization plane and non-vanishing ellipticity) allows in principle a determination of both the mass and the coupling of this particle, as well as a discrimination between the scalar and pseudoscalar cases.

We consider the classical equations of motion, obtained from the full action, i.e., the Maxwell and Klein-Gordon actions for the electromagnetic and for the pseudoscalar field, plus the interaction given in (1). For the electromagnetic field strength, we write:

$$F_{\mu\nu} = F_{\mu\nu}^{ext} + \partial_\mu A_\nu - \partial_\nu A_\mu$$

where $F_{\mu\nu}^{ext}$ represents the external magnetic field, and A_μ the vector potential associated with the light wave. The gauge condition:

$$\vec{\nabla}\cdot\vec{A} = 0$$

will be used. For the case at hand, where propagation is orthogonal to $\vec{B}$, one can further specify[4] the condition:

$$A_0 = 0$$

Keeping only linear terms in $\vec{A}$ and ϕ, one obtains the set of coupled equations:

$$\begin{aligned} &\Box \vec{A} + \frac{1}{M}\frac{\partial\varphi}{\partial t}\vec{B} = 0 \\ &-\frac{1}{M}\vec{B}\cdot\frac{\partial\vec{A}}{\partial t} + (\Box + m^2)\varphi = 0 \end{aligned} \tag{4}$$

where m is the mass of the pseudoscalar particle. From Eqs. (4) we see that only the component of $\vec{A}$ parallel to $\vec{B}$ is affected. Thus for a wave linearly polarized in the orthogonal direction with respect to $\vec{B}$, we can set:

$$\begin{aligned} &A_\perp(t,\vec{x}) = e^{-i(\omega t - \vec{k}\cdot\vec{x})} \\ &\vec{k}\cdot\vec{B} = 0 \end{aligned} \tag{5}$$

ω and k being respectively the energy and wave number of the initial beam ($|\vec{k}| = \omega$, natural units used throughout). Photons polarized parallel to $\vec{B}$ mix with the ϕ field. We look for solutions of the form:

$$
\begin{aligned}
A_{\parallel} &= A\, e^{-i(\omega' t - \vec{k}\cdot\vec{x})} \\
\varphi &= \Phi\, e^{-i(\omega' t - \vec{k}\cdot\vec{x})}
\end{aligned}
\qquad (6)
$$

These solutions exist if ω' is a root of the secular equation derived from Eq. (4), i.e.:

$$
(k^2 - \omega'^2)(k^2 + m^2 - \omega'^2) - \frac{B^2}{M^2}\omega'^2 = 0 \qquad (7)
$$

which yields the two values:

$$
\begin{aligned}
\omega'^2 = \omega_\pm^2 &= k^2 + \frac{1}{2}\left[m^2 + \frac{B^2}{M^2} \pm \sqrt{\left(m^2 + \frac{B^2}{M^2}\right)^2 + 4\frac{k^2 B^2}{M^2}} \right] = \\
&\equiv k^2 + \delta_\pm
\end{aligned}
\qquad (8)
$$

Note that $\omega_\pm^2 > 0$ for any value of the parameters, and that $\omega_- = 0$ for vanishing k, as a consequence of the gauge invariance of the full Lagrangian. A unique solution is obtained by imposing the initial conditions:

$$
\begin{aligned}
A_{\parallel}(t=0, \vec{x}=0) &= 1 \\
\varphi(t=0, \vec{x}=0) &= 0
\end{aligned}
\qquad (9)
$$

This gives

$$
\begin{aligned}
A_{\parallel}(t,\vec{x}) &= \frac{1}{D(k)}\left[(k^2 - \omega_-^2)\omega_+ e^{-i\omega_+ t} - (k^2 - \omega_+^2)\omega_- e^{-i\omega_- t} \right] e^{-i\vec{k}\cdot\vec{x}} \\
D(k) &= (k^2 - \omega_-^2)\omega_+ - (k^2 - \omega_+^2)\omega_- \\
\omega_\pm &= +\left(\omega_\pm^2\right)^{1/2}
\end{aligned}
\qquad (10)
$$

We consider now a wave which, at t = 0, is linearly polarized at an angle α with respect to $\vec{B}$, so that:

$$\vec{A}(0) = \cos\alpha\,\vec{i} + \sin\alpha\,\vec{j} \tag{11}$$

(we omit in the following the $\vec{x}$-dependence, $\vec{i}$ and $\vec{j}$ are unit vectors parallel to $\vec{B}$ and orthogonal to $\vec{B}$ and $\vec{k}$, respectively). After a time $t = L$:

$$\vec{A}(t) = \cos\alpha\, A_{\parallel}(t)\cdot\vec{i} + \sin\alpha\, e^{-i\omega t}\cdot\vec{j} \tag{12}$$

As we discuss below, we can approximate $A_{\parallel}(t)$ with the expression:

$$A_{\parallel}(t) = [1 + \epsilon(L) + i\varphi(L)]\, e^{-i\omega t} \tag{13}$$

ε and ϕ being very small quantities. To lowest order in ε and ϕ, Eqs. (12) and (13) represent a wave which is elliptically polarized. The vector potential describes an ellipse with the major axis at an angle

$$\alpha(L) = \alpha - \frac{1}{2}\,\epsilon(L)\sin 2\alpha \tag{14}$$

with respect to $\vec{B}$. The ellipticity e (equal to the ratio of the minor to the major axis) is:

$$e = \frac{1}{2}\,|\varphi(L)| \tag{15}$$

Notice that the net rotation, $\alpha(L)-\alpha$, depends upon the sign of α, which changes if we flip $\vec{k}$ for fixed $\vec{B}$. Therefore, one cannot accumulate the rotation effect by making the light to travel back and forth in the magnet. On the other hand, the ellipticity does accumulate.

To find ε and ϕ we have to distinguish different regimes, characterized by a rapid or a slow variation of the phase factors appearing in the right-hand side of Eq. (10), with respect to the phase factor $e^{-i\omega t}$ of the orthogonal component. In the region of rapid variation, the phase factor averages to zero, and the corresponding term drops out. We have three different regions:

i) $|\omega_{\pm}-\omega| L \ll 2\pi$. In this case $\omega_{\pm}$ are extremely near to ω, and both the $A_{\parallel}$ and the ϕ waves propagate coherently with $A_{\perp}$ over the whole length L. By expanding $\omega_{\pm}$ around ω, one finds:

$$\epsilon(L) \simeq -\frac{B^2L^2}{8M^2}$$

$$\varphi(L) \simeq \frac{B^2m^2}{48K^2M^2}L^3 \tag{16}$$

The boundary of this region can be (somewhat arbitrarily) fixed at:

$$|\omega_+ - \omega| L = 2\pi \tag{17}$$

where the wave frequency ω_+ ceases to propagate coherently. Translated into a condition for the parameters m and M, Eq. (17) reads:

$$m^2 = m_o^2 \left[1 - \left(\frac{M_o}{M}\right)^2\right] \tag{18}$$

where

$$m_o^2 = \frac{4\pi k}{L}$$

$$M_o = \frac{BL}{4\pi} \tag{19}$$

ii) $\left|\omega_+ - \omega\right| L \gg 2\pi \gg \left|\omega_- - \omega\right| L$. In this region, only the wave with frequency ω_- propagates coherently. By expanding ω_- around ω in the coherent term of Eq. (10), one finds:

$$\epsilon(L) = \frac{\omega_+ \delta_-}{K\delta_+} \tag{20}$$

$$\varphi(L) = L(k - \omega_-) = -\frac{L\delta_-}{2K} \tag{21}$$

with $\delta_\pm$ defined in Eq. (8).

iii) $\left|\omega_\pm - \omega\right| L \gg 2\pi$. In this region, $A_\parallel$ is completely incoherent, and transformed into a mixture of photons and pseudoscalar particles. The boundary of this region is given by the equation:

$$m^2 = m_o^2 \left[\left(\frac{M_o}{M}\right)^2 - 1\right] \tag{22}$$

This region corresponds to completely unrealistic values of M, i.e., $M < M_0$ (see below).

We discuss now the experimental conditions. Foreseeable values[2)] of B, L and ω are:

$$B = 10^5 \text{ Gauss}$$
$$L = 10^5 \text{ cm} \tag{23}$$
$$\omega = 2.4 \text{ eV}$$

Accordingly, one obtains:

$$m_0 \approx 7.7 \times 10^{-5} \text{ eV}$$
$$M_0 = 2.8 \cdot 10^3 \text{ GeV} \tag{24}$$

The quantity m_0 sets the scale for the range of masses m to which the experiment is sensitive. Indeed, as can be seen from the previous formulae, all effects vanish rapidly for $m \gg m_0$. For example, the known pseudoscalar mesons, π^0 and η^0, give unmeasurably small contributions. Analogously, M_0 sets the scale of sensitivity for M except that, since the achievable sensitivity is very large and the effects do not drop for $M \gg M_0$ as fast as they do for large m, considerably larger values of M can still be explored.

As for the sensitivity, measurements of ϕ as small as:

$$\varphi = 10^{-12} \text{ rad} \tag{25}$$

seem to be attainable[2)]. By suitable modification of the apparatus proposed in Ref. 2), one may hope to obtain the same level for the measurement of the rotation in the polarization plane, η:

$$\eta(L) = \alpha(L) - \alpha \tag{26}$$

In the figure we display the curves of constant ϕ and η corresponding to

$$\varphi = \eta = 10^{-12} \text{ rad} \tag{27}$$

and with the initial condition $\sin 2\alpha = 1$. In the same figure, the boundaries of the different regions, corresponding to Eqs. (18) and (22), are reported.

A non-vanishing signal for $\phi(L)$ is expected anyhow, arising from photon-photon scattering induced by an electron loop. Using the well-known Euler-Heisenberg effective Lagrangian[8)] one finds:

$$\varphi_{QED} = \frac{\alpha^2}{30\pi} \cdot \frac{L\omega B^2}{m_e^4} =$$

$$= \frac{2}{15}\alpha^2 \frac{(M_o m_o)^2}{m_e^4} \simeq 5.\cdot 10^{-12} \tag{28}$$

The presence of an almost massless pseudoscalar particle with $M \approx 10^{10}$ GeV would manifest itself with a sizeable deviation from the pure QED prediction. No conventional background to $\eta(L)$ is expected.

It is interesting to discuss the possibility to detect in this way an invisible axion (3b) [the detection of cosmic axions based on the coupling in Eq. (1) has been discussed in Ref. 9)]. For this particle one expects a relation between mass m and symmetry breaking vacuum expectation value of the form:

$$mV = R\, m_\pi f_\pi \tag{29}$$

with R a model-dependent coefficient. Since we expect from Eq. (2):

$$M \simeq 10^3 V$$

we obtain

$$m \simeq R \left(\frac{10^{10}\,\mathrm{GeV}}{M}\right) \cdot 1\,\mathrm{eV} \tag{30}$$

It is clear from the figure that $R \approx 1$ and $M \approx 10^{10}$ GeV would give a mass too large for a detectable effect. Measurable effects result for $R \lesssim 10^{-3}$. On the other hand a truly Goldstone boson would be fully detectable, for M in this range.

Lower bounds to M from astrophysical considerations have been widely discussed in the literature[5)-7)]. The bound arises because a too large coupling in Eq. (1) would give rise to an unacceptably large star cooling, via the Primakoff process:

$$\gamma + Z \rightarrow \varphi + Z \tag{31}$$

mediated by one-photon exchange. The most recent discussion[7)] gives the bound:

$$M > 3 \cdot 10^{8}\ \mathrm{GeV} \tag{32}$$

which is considerably smaller than the values considered in Fig. 1. Larger bounds for V arise from cooling via the bremsstrahlung process

$$e + Z \to e + \varphi + Z \tag{33}$$

However, this bound depends on the $\phi - e - \bar{e}$ coupling which is, a priori, independent of the coupling in Eq. (1).

Finally, we comment on the effect induced by a scalar particle through the coupling in Eq. (3). It is not difficult to see that, to leading order, everything is equal to the previous case but for an exchange of the parallel with the orthogonal components of the electromagnetic field. In turn, this simply amounts to a change of sign for the net rotation of the polarization plane, Eq. (26).

In conclusion, we have shown that a precise measurement of the change in the polarization state of a photon travelling in a magnetic field can give significant limits or determine the coupling, mass and parity and any hypothetical, light, spinless particle coupled to two photons.

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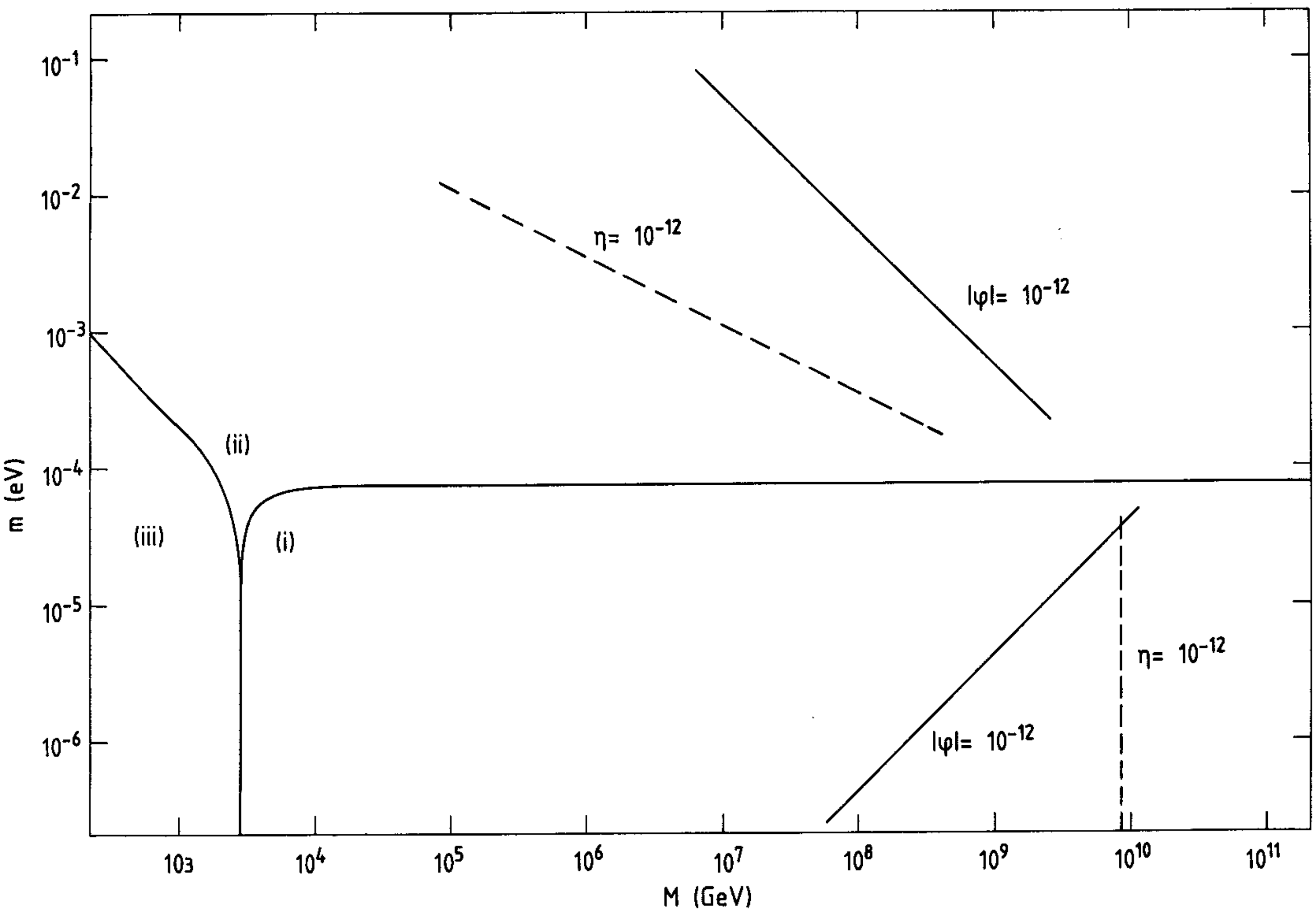
FIGURE CAPTION

Lines corresponding to a given ellipticity (solid) and to a given rotation of the polarization plane (dashed), in the m-M plane. The regions (i) to (iii) correspond to the three different coherence regimes discussed in the text. The initial beam is assumed to be linearly polarized at 45° with respect to the magnetic field.



m (eV)
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
M (GeV)
10³
10⁴
10⁵
10⁶
10⁷
10⁸
10⁹
10¹⁰
10¹¹
(i)
(ii)
(iii)
η= 10⁻¹²
|φ|= 10⁻¹²
|φ|= 10⁻¹²
η= 10⁻¹²